

# Optimal Bilateral Cooperative Slot Allocation for Two Liner Carriers under a Co-Chartering Agreement

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The container liner shipping industry has stepped into an era of international strategic alliances. Important to these liner alliances is the sharing and allocation of container slots between its member carriers. This paper optimises planning of container ship capacity sharing and co-allocation under a co-chartering agreement. First, we explain the concept of this business agreement and its implications on maritime operations. Then, we identify key influencing factors that may affect the decisions of cooperative slot co-allocation. The slot co-allocation problem is modelled as an Integer Programming problem and solved using data from two routes between the United States and Asia. The model determines the optimal slot co-allocation strategies between shipping alliance carriers along allied shipping routes. Computational results indicate that the proposed method is effective in obtaining optimal, cooperative slot sharing strategies that can maximise the total system revenue.

## KEY WORDS

1. Container. 2. Slot co-allocation. 3. Liner shipping. 4. Co-chartering.

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1. INTRODUCTION. Containerised liner shipping is the main conduit of international trade, moving over 95% of manufactured goods worldwide (Fransoo and Lee, 2013). In pace with the growth of the global economy, the shipping industry is undergoing a transformation by introducing larger ships and globalising shipping networks (Panayides and Cullinane, 2002). Currently, about 15 ocean carriers hold a dominant share of the international containerised liner shipping market (UNCTAD, 2015). Cooperation under alliances

formed through cooperative agreements has emerged as a major characteristic of recent shipping services (Yap, 2014).

Slot co-chartering is a common business practice within shipping alliances. Under this cooperative agreement, different carriers share slot capacity on similar routes. The extent of capacity sharing and lease/rental cost are detailed in the so-called co-chartering agreement (Caschili et al., 2014). The success of this cooperative business agreement relies on a seamless synergistic arrangement (Tran and Haasis, 2013). For each individual carrier, there are at least two key questions that this arrangement must answer, including:

- How many slots should be allocated to each carrier in a cooperative round voyage?
- What is the best slot allocation strategy for different container types shipped between different port pairs on the shipping routes?

In recognition of their inherent importance, this research is intended to develop novel operation models to optimally address the above-mentioned questions. In the long run, this research could aid liner carriers by evolving into a synergy management framework so that they can share their ship slot resource optimally. Specifically, this paper extends the literature of slot allocation management by modelling slot allocation under a co-chartering agreement.

2. LITERATURE REVIEW. Container shipping alliances originate from the late 1990s, flourishing in the early 21st century (Wang, 2015). These alliances are widely seen on major global maritime routes, connecting Asia, Europe and America (Lewandowski, 2015).

Some studies have focused on container liner shipping management and shipping alliances (Ryoo and Thanopoulou, 1999; Midoro and Pitto, 2000; Slack et al., 2002; Song and Panayides, 2002; Lu et al., 2006; Panayides and Cullinane, 2002). Many similar studies of shipping alliances have focused on maritime economics, management, and policy (Benacchio et al., 2007; Yang et al., 2011; Gao and Yoshida, 2013; Parola et al., 2015). Most contemporary studies on container slot allocation are related to revenue management (Meng et al., 2013; Ting and Tzeng, 2004; Feng and Chang, 2008 and Zurheide and Fischer, 2012). Other recent methods have been developed to understand the mechanisms involved in shipping alliances and the optimal business decisions within an alliance (Agarwal and Ergun, 2010; Wu, 2012; Dong et al., 2015; Zheng et al., 2015).

Previous studies mainly considered a single carrier's shipping service without accounting for container slot sharing and co-allocation. Other past studies investigated the shipping alliance problem through a "macro" approach, such as shipping network design, capacity allocation mechanism design and economic policy. However, they did not account for specific agreements for slot sharing among alliance members. Lastly, some scholars considered some business strategies such as slot purchase (Lu et al., 2010) and joint fleet (Chen and Yahalom, 2013). However, no prior research, including these, has yet considered slot co-chartering as a means to share capacity. Slot co-chartering is a more complex and flexible business agreement and thus, needs further research.

3. PROBLEM STATEMENT. Under a mutual slot co-chartering agreement, two liner carriers will collaboratively allocate their pool of available slots to maximize total system revenue. Because each alliance member has equal negotiation rights when developing slot

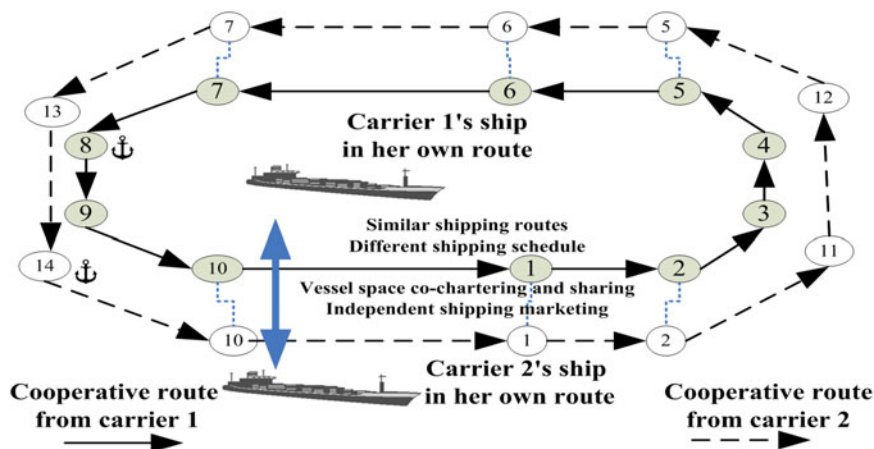


Figure 1. Slot co-chartering and co-allocation on cooperative shipping routes.

sharing contracts, the decisions that yield the maximum total slot-related revenue for the system also maximize each participating member's revenue.

3.1. *Slot co-allocation with a co-chartering agreement.* Besides slot co-chartering, there are other different cooperative modes of ship slot sharing. Among these slot sharing modes, slot co-chartering emphasises the bilateral cooperation relationship between two carriers. The two carriers have a bilateral slot buyer-seller relationship. The purchasing and selling actions of each member occur simultaneously. Because of differing slot values for different voyages, the alliance members need to deliberately negotiate the slot rent/lease fee.

As illustrated in Figure 1 (Nodes 1 to 14 represent calling ports), under a slot co-chartering agreement, the two alliance members can mutually rent the other's slots on two similar cooperative routes. The two alliance members settle slot rent agreements according to different complete voyage periods. Slot co-chartering means that the two carriers mutually rent to each other a certain number of slots and reefer plugs (but not necessarily an equal number of slots per carrier) and that the two carriers settle slot rents and additional refrigerated storage costs.

3.2. *Key factors affecting slot co-allocation.* The following factors affect slot co-allocation decisions of alliance members. The first factor is slot cost, which represents the variable cost for transporting a single container. The second factor is freight rate, which is used to calculate the revenue standard corresponding to different types of container and transportation between different port of call pairs. The third factor is the shipping demand, which is the actual volume of container cargo that a ship carries between specific port pairs. The fourth factor is the type of container (i.e. size, capability, and whether it is empty or full).

4. MODEL FORMULATION. This section proposes an optimisation model for container slot co-allocation under a co-chartering agreement between alliance members.

4.1. *Some considerations.* Accounting for the actual business operation of liner alliances, the following considerations are incorporated in the model:

- Slots are shared among relevant voyages and containers in ports between alliance trunk routes. Container freight demands at each hub port on trunk lines can be accomplished in a timely and effective manner.
- Given an alliance slot sharing agreement, the two cooperative alliance routes are known. The ports of call and the sequence of the ports of call on the two different alliance routes are known. The two specific vessels are on two known routes maintained by two different members.
- When the two cooperative shipping routes are known, liner service time and frequency of two shipping companies will be relatively stable. Therefore, time factor has little effect on slot co-allocation decisions.
- The slot rent fees for the two cooperative voyages are known. The additional costs of the reefer plugs for rent and lease between alliance members are also known. The available shipping capacities of the specific ship for each voyage that alliance members operate are known. The maximum limit of the voyage slots for rent and sharing between the two members are known.
- The freight rate and voyage costs between the port pairs of origin and destination are determined in advance for each member. We can set the maximum and minimum limits for the demands of voyage port pairs of origin and destination according to each member's local freight agent's historical demand data.
- The containers of alliance members cannot be overweight and must conform to the loading safety requirements. The model mainly considers available slot capacity and the special requirements for reefer plugs. Container slot allocation does not violate the total deadweight safety requirement.

4.2. *Slot co-allocation planning model.* In the following sections, the indices, sets, parameters, and decision variables of the container slot co-allocation planning model are defined. The model aims to maximise total slot-related revenue gained by the co-chartering and allocation of the ship space on the corresponding round-trip voyages.

4.2.1. *Indices.* The following indices are defined.  $Z$  is the slot-related revenue of the shipping alliance system on two cooperative shipping routes.  $i, j$  are shipping alliance members and  $(i, j)$  represents a pair of members.  $o, d$  denotes a port and  $(o, d)$  denotes a pair of ports on one segment of the route.  $k$  is a container type, twelve different kinds of laden and empty container are considered,  $k$  may be 20GP (general-purpose containers), 40GP, 20RF (refrigerated containers), 40RF, 20OT (open top containers), 40OT, each of which may be laden or empty.  $r$  is the route category and may be self-operated routes by one member or cooperative routes.  $s$  is a route segment category on one route.

4.2.2. *Sets.*  $SA$  is the set of alliance members,  $\forall i, j \in SA, i \neq j$ .  $R$  is the set of alliance routes. Self-operated and cooperative routes are collectively known as alliance routes.

$$R = \bigcup_{i \in SA} R_i = \bigcup_{\substack{i, j \in SA \\ i \neq j}} (R_i \cup R_j), \forall i, j \in SA, i \neq j \text{ where } R_i/R_j \text{ is a set of routes operated}$$

by member  $i/j$ .  $CO$  is a set of all container types. We define  $CO_{20}$  as the set of 20-foot containers, and  $CO_{40}$  as the set of 40-foot containers respectively. So  $CO = CO_{20} \cup CO_{40} = CT \cup EC$ , where  $CT$  is the set of all laden container types and  $EC$  is the set of all empty container types.

$RF$  is the set of refrigerated laden containers including 20RF and 40RF laden ones.  $\Omega^r$  is the set of all origin-destination port pairs on alliance route  $r$ .  $\Omega^r = \{(o, d) | r \in R\}$  and  $S^r$  is the set of voyage segments on alliance route  $r$ .

4.2.3. *Parameters.*  $a_{od}^{rs}$  represents whether the container delivery passage of port pair  $(o, d)$  passes leg  $s$  ( $s \in s^r$ ), 1 for yes, 0 otherwise.  $t^k$  is a convert coefficient of container types to represent number of slots occupied in Twenty-foot Equivalent Unit (TEU) per container of category  $k$ , if  $k \in CO_{20}, t^k = 1$ ; if  $k \in CO_{40}, t^k = 2$ .  $F_{iod}^{rk}$  is the freight revenue of each  $k$ -type container delivered between port pair  $(o, d)$  on route  $r$  for member  $i$ . When  $k \in EC, F_{iod}^{rk} = 0, \forall r \in R, \forall i \in SA, \forall (o, d) \in \Omega^r$ .  $C_{iod}^{rk}$  is the variable costs of each  $k$ -type container delivered between port pair  $(o, d)$  on route  $r$  for member  $i$ .  $\forall i \in SA, \forall r \in R, \forall (o, d) \in \Omega^r$ .  $B_{ij}^r$  is the slot rent which member  $i$  should pay for renting from member  $j$  on alliance route  $r$  ( $r \in R_j$ ).  $\forall r \in R_j, \forall i, j \in SA, i \neq j$ .  $D_{ij}^r$  is the slot revenue which member  $i$  can expect from leasing per TEU to member  $j$  on alliance route  $r$  ( $r \in R_i$ ).  $\forall r \in R_i, \forall i, j \in SA, i \neq j$ .  $BS_{ij}^r$  is the additional cost the member  $i$  should pay for renting one reefer plug from member  $j$  on alliance route  $r$  ( $r \in R_j$ ).  $\forall r \in R_j, \forall i, j \in SA, i \neq j$ .  $DS_{ij}^r$  is additional revenue which the member  $i$  can expect from leasing one reefer plug to member  $j$  on alliance route  $r$  ( $r \in R_j$ ).  $\forall r \in R_i, \forall i, j \in SA, i \neq j$ .  $U_i^r$  is available capacities in TEU deployed for member  $i$  on alliance route  $r$  ( $r \in R_i$ ). Similarly, we have  $U_j^r$ .  $RP_i^r$  is the available number of reefer plugs for member  $i$  on alliance route  $r$  ( $r \in R_i$ ). Similarly, we have  $RP_j^r$ .  $SP_j^r$  is the maximum number of rented slots available from member  $j$  on alliance route  $r$  according to the alliance agreement,  $SP_j^r < U_j^r, \forall r \in R_j, \forall j \in SA, i \neq j$ .  $SP_{fj}^r$  is the maximum number of rented reefer plugs available from member  $j$  on alliance route  $r$  according to the alliance agreement,  $SP_{fj}^r < RP_j^r, \forall r \in R_j, \forall j \in SA, i \neq j$ .  $DL_{iod}^{rk}$  is the minimum number of contracted  $k$ -type container slots available between port pair  $(o, d)$  for member  $i$  on alliance route  $r, \forall k \in CO$ .  $DU_{iod}^{rk}$  is the maximum number of contracted  $k$ -type container slots available between port pair  $(o, d)$  for member  $i$  on alliance route  $r, \forall k \in CO$  and  $EC_{iod}^{rk}$  is the repositioning demand of  $k$ -type empty containers available between port pair  $(o, d)$  for member  $i$  on cooperative route  $r. \forall k \in EC$ .

4.2.4. *Decision variables.*  $X_{iod}^{rk}$  is the number of  $k$ -type containers shipped between port pair  $(o, d)$  in the self-operated round route  $r$  ( $r \in R_i$ ) for member  $i \forall (o, d) \in \Omega^r, \forall k \in CO$ .  $Y_{iod}^{rk}$  is the number of  $k$ -type containers that member  $i$  should rent from member  $j$ , shipped between port pair  $(o, d)$  in the cooperative round-trip route  $r$  ( $r \in R_j$ ).  $\forall (o, d) \in \Omega^r, \forall k \in CO$ .  $q_{ij}^r$  is the optimal amount of TEU slots that member  $i$  should rent from member  $j$  on a cooperative round route  $r$  ( $r \in R_j$ ).  $p_{ij}^r$  is the optimal amount of TEU slots the member  $i$  leases to member  $j$  on a cooperative round-trip  $r$  ( $r \in R_i$ ).  $qf_{ij}^r$  is the optimal amount of reefer plugs that the member  $i$  should rent from member  $j$  on a cooperative round route  $r$  ( $r \in R_j$ ) and  $pf_{ij}^r$  is the optimal amount of reefer plugs that member  $i$  should lease to member  $j$  on a cooperative round-trip route  $r$  ( $r \in R_i$ ).

4.2.5. *Model.* The slot co-allocation planning model for liner alliance carriers with co-chartering agreement is formulated as follows:

$$\begin{aligned}
 \text{Max } Z = & \sum_{\forall i \in SA} \sum_{r \in R_i} \sum_{(o,d) \in \Omega^r} \sum_{k \in CO} (F_{iod}^{rk} - C_{iod}^{rk}) X_{iod}^{rk} + \sum_{\substack{\forall i, j \in SA \\ i \neq j}} \sum_{r \in R_j} \sum_{(o,d) \in \Omega^r} \sum_{k \in CO} (F_{iod}^{rk} - C_{iod}^{rk}) Y_{iod}^{rk} \\
 & - \sum_{\forall i \in SA} \sum_{\forall j \in SA, j \neq i} \sum_{r \in R_j} q_{ij}^r B_{ij}^r + \sum_{\forall i \in SA} \sum_{\forall j \in SA, j \neq i} \sum_{r \in R_j} p_{ij}^r D_{ij}^r - \sum_{\forall i \in SA} \sum_{\forall j \in SA, j \neq i} \sum_{r \in R_j} qf_{ij}^r BS_{ij}^r \\
 & + \sum_{\forall i \in SA} \sum_{\forall j \in SA, j \neq i} \sum_{r \in R_j} pf_{ij}^r DS_{ij}^r
 \end{aligned} \tag{1}$$

$$\sum_{(o,d) \in \Omega^r} \sum_{k \in CO} a_{od}^{rs} t^k x_{iod}^{rk} \leq U_i^r - \sum_{j \in SA} P_{ij}^r \quad \forall r \in R_i, \forall s \in S^r, \forall i, j \in SA, i \neq j \tag{2}$$

$$\sum_{(o,d) \in \Omega^r} \sum_{k \in RF} a_{od}^{rs} x_{iod}^{rk} \leq RP_i^r - \sum_{j \in SA} Pf_{ij}^r \quad \forall r \in R_i, \forall s \in S^r, \forall i, j \in SA, i \neq j \tag{3}$$

$$\sum_{(o,d) \in \Omega^r} \sum_{k \in CO} a_{od}^{rs} t^k Y_{ijod}^{rk} \leq \sum_{j \in SA} q_{ij}^r \quad \forall r \in R_j, \forall s \in S^r, \forall i, j \in SA, i \neq j \tag{4}$$

$$\sum_{(o,d) \in \Omega^r} \sum_{j \in SA} \sum_{k \in RF} a_{od}^{rs} Y_{ijod}^{rk} \leq \sum_{j \in SA} qf_{ij}^r \quad \forall r \in R_j, \forall s \in S^r, \forall i, j \in SA, i \neq j \tag{5}$$

$$\sum_{i \in SA} p_{ij}^r \leq SP_j^r \quad \forall r \in R_j, \forall i, j \in SA, i \neq j \tag{6}$$

$$\sum_{i \in SA} pf_{ij}^r \leq SPf_j^r \quad \forall r \in R_j, \forall i, j \in SA, i \neq j \tag{7}$$

$$q_{ij}^r = p_{ij}^r \quad \forall r \in R_j, \forall i, j \in SA, i \neq j \tag{8}$$

$$qf_{ij}^r = pf_{ij}^r \quad \forall r \in R_j, \forall i, j \in SA, i \neq j \tag{9}$$

$$DL_{iod}^{rk} \leq X_{iod}^{rk} + \sum_{j \in SA} Y_{jiod}^{rk} \leq DU_{iod}^{rk} \quad \forall r \in R_i, \forall k \in CT, \forall i, j \in SA, i \neq j, \forall (o, d) \in \Omega^r \tag{10}$$

$$X_{iod}^{rk} + \sum_{j \in SA} Y_{jiod}^{rk} \leq EC_{iod}^{rk} \quad \forall r \in R_i, \forall k \in EC, \forall i, j \in SA, i \neq j, \forall (o, d) \in \Omega^r \tag{11}$$

$$X_{iod}^{rk} \in N \cup \{0\} \quad \forall i \in SA, r \in R, k \in CO, (o, d) \in \Omega^r \tag{12}$$

$$Y_{iod}^{rk} \in N \cup \{0\} \quad \forall i \in SA, r \in R, k \in CO, (o, d) \in \Omega^r \tag{13}$$

$$q_{ij}^r \in N \cup \{0\} \quad \forall i, j \in SA, i \neq j, \forall r \in R_j \tag{14}$$

$$p_{ij}^r \in N \cup \{0\} \quad \forall i, j \in SA, i \neq j, \forall r \in R_i \tag{15}$$

$$qf_{ij}^r \in N \cup \{0\} \quad \forall i, j \in SA, i \neq j, \forall r \in R_j \tag{16}$$

$$pf_{ij}^r \in N \cup \{0\} \quad \forall i, j \in SA, i \neq j, \forall r \in R_i \tag{17}$$

The objective function Equation (1) aims to maximise total slot-related revenue from various container categories for the shipping alliance. Equation (2) confines the capacity of the ships involved. The slots occupied by the containers from the alliance members should not exceed the available number of slots for each party on the specific running vessel on any segment of the cooperative shipping route. Equation (3) the available vessel refrigerated capacities on alliance routes. Equations (4) to (7) define the rental limit set for the slots and reefer plugs. Generally speaking, a maximum limit of slots and reefer plugs will be set for each party in the agreement. Equations (8) and (9) define the symmetry constraint for slot rental and leasing of alliance members. One alliance member rents slots and correspondingly the other leases them. Similarly, symmetry exists between the reefer plug rental and leasing. Equation (10) represents the cargo demand constraint for laden container slots. Equation (11) specifies a constraint on the demand for empty containers. This constraint requires carriers to satisfy the possible need for shipping empty containers



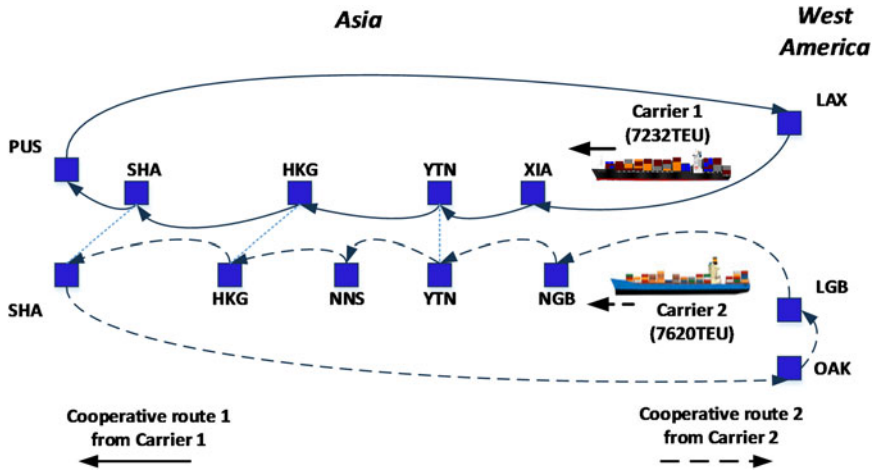


Figure 2. Cooperative container shipping service with a co-chartering agreement between two liner carriers.

from surplus areas to shortage areas on a specific voyage. Equations (12) to (17) are non-negative integer constraints.

Slot co-allocation in a liner alliance can be modelled as an Integer Programming (IP) problem, in which all of the variables are integers. In addition, the objective function and the constraints are linear. Integer programming is NP-hard (Papadimitriou, 1981). Several algorithms, such as branch and bound (Narendra and Fukunaga, 1977; Kuno, 2002; Conforti et al., 2014), have been implemented in commercial solvers to address the integer programming problem.

**5. NUMERICAL EXPERIMENTS.** This section uses an example of two cooperative shipping routes between the West Coast of the United States and China, under a slot co-chartering agreement between two liner shipping carriers (Figure 2). According to a non-disclosure agreement with our data suppliers, we make the two carriers anonymous in this paper. The numerical example uses data from November 2012. The methodology can be readily adapted when future data become available.

**5.1. Background for the studied routes.** We consider two similar West Coast American-Asia cooperative shipping routes, route 1 and route 2. Route 1 is operated by Carrier 1, and route 2 is operated by Carrier 2. As shown in Figure 2, the ports of call for route 1 are: LAX (Los Angeles) - XIA (Xiamen) - YTN (Yantian) - HKG (Hong Kong) - SHA (Shanghai) - PUS (Pusan) - LAX (Los Angeles). Similarly, the ports of call for shipping route 2 are: OAK (Auckland) - LGB (Long Beach) - NGB (Ningbo) - YTN (Yantian) - NNS (Nansha) - HKG (Hong Kong) - SHA (Shanghai) - OAK (Auckland).

There are two round voyages in this example. For a round voyage on shipping route 1, the available space for leasing to another carrier on Carrier 1's ship is 7232 TEUs (with a total of 500 reefer plugs available for reefer containers). For a similar round-trip voyage on route 2, the available container slot space on Carrier 2's ship is 7620 TEUs (with 520 reefer plugs available). According to the slot co-chartering agreement between the two liner carriers, Carrier 1 offers a maximum of 3000 TEUs with a maximum of 300 reefer plugs to Carrier 2 on route 1. Similarly, Carrier 2 will lease a maximum of 3000 TEUs and 310

Table 1. Relationship between port pairs and legs ( $a_{od}^s$ ) for the cooperative route 1.

port pairs	Sailing legs (s)						port pairs	Sailing legs (s)					
	1-2	2-3	3-4	4-5	5-6	6-1		1-2	2-3	3-4	4-5	5-6	6-1
1-2	1	0	0	0	0	0	4-5	0	0	0	1	0	0
1-3	1	1	0	0	0	0	4-6	0	0	0	1	1	0
1-4	1	1	1	0	0	0	4-1	0	0	0	1	1	1
1-5	1	1	1	1	0	0	4-2	1	0	0	1	1	1
1-6	1	1	1	1	1	0	4-3	1	1	0	1	1	1
2-3	0	1	0	0	0	0	5-6	0	0	0	0	1	0
2-4	0	1	1	0	0	0	5-1	0	0	0	0	1	1
2-5	0	1	1	1	0	0	5-2	1	0	0	0	1	1
2-6	0	1	1	1	1	0	5-3	1	1	0	0	1	1
2-1	0	1	1	1	1	1	5-4	1	1	1	0	1	1
3-4	0	0	1	0	0	0	6-1	0	0	0	0	0	1
3-5	0	0	1	1	0	0	6-2	1	0	0	0	0	1
3-6	0	0	1	1	1	0	6-3	1	1	0	0	0	1
3-1	0	0	1	1	1	1	6-4	1	1	1	0	0	1
3-2	1	0	1	1	1	1	6-5	1	1	1	1	0	1

Note: The port-pair numbers on all tables indicate the following ports: 1 = LAX (Los Angeles), 2 = XIA (Xiamen), 3 = YTN (Yantian), 4 = HKG (Hong Kong), 5 = SHA (Shanghai), 6 = PUS (Pusan), 7 = OAK (Oakland), 8 = LGB (Long Beach), 9 = NGB (Ningbo), 10 = NNS (Nansha).

reefer plugs to Carrier 1 for a round trip on shipping route 2. On route 1, slot rental costs \$300 per TEU and each rented reefer plug is \$340. On route 2, the rental fee is \$250 per TEU and \$360 per reefer plug.

Liner carriers transport standard types of 20-ft containers, some 40-ft containers, and some special types for “out-of-gauge” cargo such as 20-ft and 40-ft OT and RF containers. To simplify the problem, these out-of-gauge containers are described as  $t^k$ . As previously defined, if  $k \in CO_{20}$ , then  $t^k = 1$ , if  $k \in CO_{40}$ , then  $t^k = 2$ , which means that a 40-ft container requires two slots for transportation. The parameter (1 and 0) defines whether a sailing leg is on a specific delivery passage between the port pair (1 = yes, 0 otherwise) (Table 1 and Table 2). For example, on route 1, from port 1 to port 5, four sailing legs are involved (1-2, 2-3, 3-4, 4-5). There are 6 sailing legs and 30 port pairs on shipping route 1, and 7 sailing legs and 42 port pairs on shipping route 2.

5.2. Results Analysis and Discussion. The model has 1736 integer variables and 1360 constraints. A commercial optimisation software program named LINGO 11.0 was used in solving the problem. The experiment environment utilised an Intel Core i7-4710HQ CPU, quad-core 2.50 GHz, 8.00 GB memory, and the Win8.1 64-bit operating system. LINGO 11.0 obtained the optimal solutions after 44,960 iterations in this experimental environment (computation time: 29 seconds). The optimal slot co-chartering policy is as follows: on the cooperative voyage along route 1, Carrier 1 should lease 3000 TEUs and 205 reefer plugs to Carrier 2; on the cooperative voyage along route 2, Carrier 2 should lease 2793 TEUs and 246 reefer plugs to Carrier 1. Table 3 presents the detailed optimal results.

Table 3 shows the optimal slot co-allocation results in TEU for all sailing legs of the two cooperative shipping routes. In Table 3, Carrier 1 and Carrier 2 both share each other’s slot resources to transport laden containers and empty containers on the two cooperative routes. Carrier 1’s laden containers carried on major segments of self-operated route 1 basically maintains a range from 4000 TEU to 4200 TEU, and only 2400 TEU to 2700



Table 2. Relationship between port pairs and legs ( $a_{od}^s$ ) for the cooperative route 2.

port pairs	Sailing legs (s)							port pairs	Sailing legs (s)						
	7-8	8-9	9-3	3-10	10-4	4-5	5-7		7-8	8-9	9-3	3-10	10-4	4-5	5-7
7-8	1	0	0	0	0	0	0	3-7	0	0	0	1	1	1	1
7-9	1	1	0	0	0	0	0	3-8	1	0	0	1	1	1	1
7-3	1	1	1	0	0	0	0	3-9	1	1	0	1	1	1	1
7-10	1	1	1	1	0	0	0	10-4	0	0	0	0	1	0	0
7-4	1	1	1	1	1	0	0	10-5	0	0	0	0	1	1	0
7-5	1	1	1	1	1	1	0	10-7	0	0	0	0	1	1	1
8-9	0	1	0	0	0	0	0	10-8	1	0	0	0	1	1	1
8-3	0	1	1	0	0	0	0	10-9	1	1	0	0	1	1	1
8-10	0	1	1	1	0	0	0	10-3	1	1	1	0	1	1	1
8-4	0	1	1	1	1	0	0	4-5	0	0	0	0	0	1	0
8-5	0	1	1	1	1	1	0	4-7	0	0	0	0	0	1	1
8-7	0	1	1	1	1	1	1	4-8	1	0	0	0	0	1	1
9-3	0	0	1	0	0	0	0	4-9	1	1	0	0	0	1	1
9-10	0	0	1	1	0	0	0	4-3	1	1	1	0	0	1	1
9-4	0	0	1	1	1	0	0	4-10	1	1	1	1	0	1	1
9-5	0	0	1	1	1	1	0	5-7	0	0	0	0	0	0	1
9-7	0	0	1	1	1	1	1	5-8	1	0	0	0	0	0	1
9-8	1	0	1	1	1	1	1	5-9	1	1	0	0	0	0	1
3-10	0	0	0	1	0	0	0	5-3	1	1	1	0	0	0	1
3-4	0	0	0	1	1	0	0	5-10	1	1	1	1	0	0	1
3-5	0	0	0	1	1	1	0	5-4	1	1	1	1	1	0	1

Table 3. Slot occupation for all sailing legs for each member (TEU units).

Route	Legs	Carrier 1		Carrier 2	
		Laden	Empty	Laden	Empty
Route 1	1-2	4093	139	2870	130
	2-3	4083	149	2870	130
	3-4	4114	118	2873	127
	4-5	4144	88	2889	111
	5-6	4186	46	2915	85
	6-1	4093	139	2930	70
Route 2	7-8	2429	364	4777	50
	8-9	2517	276	4747	80
	9-3	2565	228	4696	131
	3-10	2627	166	4633	194
	10-4	2627	166	4696	131
	4-5	2658	135	4752	75
5-7	2551	242	4777	50	

TEU on route 2, operated by Carrier 2. However, alliance slot co-chartering plays a great role in empty container transportation. Carrier 1's empty containers transported on each segment of self-operated route 1 generally fall within a range of 45 TEU to 150 TEU, while accounting for 135 TEU to 365 TEU on route 2. Carrier 2 applies a similar policy in the model, transporting its own laden containers primarily on self-operated route 2. Carrier

Table 4. Slot occupation results for all sailing legs for each member (by number of containers).

R	T	LG	Carrier 1						Carrier 2					
			20	40	20	40	20	40	20	40	20	40	20	40
			GP	GP	RF	RF	OT	OT	GP	GP	RF	RF	OT	OT
R1	L	1-2	1708	1019	253	42	10	0	0	888	28	177	200	256
		2-3	1244	1116	29	266	46	0	0	679	112	93	320	447
		3-4	859	1386	151	144	44	0	0	540	67	138	400	525
		4-5	800	1373	36	259	44	0	335	361	110	95	400	566
		5-6	1384	1122	78	217	46	0	0	615	29	176	320	492
		6-1	1773	917	114	181	10	0	0	916	34	171	200	216
	E	1-2	26	45	6	4	3	3	25	36	11	5	6	3
		2-3	28	47	6	5	3	4	25	37	13	4	6	2
		3-4	23	36	6	4	3	3	24	39	10	4	5	1
		4-5	17	28	3	3	2	2	22	35	10	2	3	1
		5-6	8	17	0	1	0	1	18	26	9	1	2	1
		6-1	26	45	6	4	3	3	15	20	9	1	2	1
R2	L	7-8	1878	0	132	114	191	0	719	1203	2	272	180	463
		8-9	2256	1	245	1	12	0	419	1355	8	266	330	374
		9-3	2085	22	243	3	187	0	38	1484	12	262	240	457
		3-10	2052	11	244	2	305	0	60	1426	39	235	180	516
		10-4	1958	11	210	36	365	0	77	1403	7	267	120	576
		4-5	1691	114	120	126	367	0	25	1545	9	265	60	519
	E	5-7	1840	1	95	151	312	0	93	1666	4	270	0	404
		7-8	70	109	24	11	12	9	10	15	5	2	1	0
		8-9	53	82	20	8	9	7	17	26	6	2	1	0
		9-3	45	69	17	6	6	5	25	39	9	5	5	2
		3-10	33	51	13	4	4	3	38	58	14	6	6	4
		10-4	32	50	13	4	5	4	26	40	10	4	3	2
		4-5	26	42	11	3	4	2	16	24	6	2	1	0
		5-7	47	74	16	6	7	6	10	15	5	2	1	0

Note: R stands for shipping route; T stands for container type; L stands for laden container; E stands for empty container; LG stands for sailing legs.

2’s laden containers account for 4600 TEU to 4800 TEU of capacity on route 2 segments; Carrier 1’s laden containers on self-operated route 1 account for 2800 TEU to 3000 TEU.

Table 4 shows the cargo flow distribution of different Carrier 1 and Carrier 2 containers on different sailing legs of route 1 and route 2. The slots of laden containers and empty containers are mainly in the type of GP containers because of the high demand for this slot type. As shown in Table 4, the volume of 20-foot and 40-foot GP containers on the two cooperative routes is rather large for both companies. As for RF and OT containers, the market demands for these two companies vary according to route. For Carrier 1, larger volumes of 20-foot and 40-foot RF containers are transported on route 1 and route 2, while there is a relatively smaller demand for OT containers on route 1. Carrier 1 rents a portion of slots for 20-foot OT containers from Carrier 2 on route 2.

According to Tables 5 and 6, three large ports on the American west coast (LAX, OAK, and LGB), are responsible for a great deal of transoceanic container transportation activity. Carrier 1 and Carrier 2 allocate a large number of container slots to these three ports to satisfy transportation demand. LAX, OAK, and LGB, as the most important hub nodes in North America and the transport corridor to Asia, have a large volume of transportation

Table 5. Number of containers allocated for each member at the ports of the cooperative routes.

R	C	P	Laden						Empty						
			20	40	20	40	20	40	20	40	20	40	20	40	
			GP	GP	RF	RF	OT	OT	GP	GP	RF	RF	OT	OT	
R1	C1	LAX	1564	890	200	0	0	0	0	0	0	0	0	0	0
		XIA	76	151	0	259	40	0	5	8	0	1	0	1	
		YTN	78	411	139	1	1	0	1	2	0	0	0	0	
		HKG	595	484	15	197	2	0	2	4	0	0	0	0	
		SHA	755	226	43	0	3	0	0	3	0	0	0	0	
	PUS	649	44	48	85	4	0	23	36	6	4	3	3		
	C2	LAX	0	888	23	166	200	240	23	36	8	5	5	3	
		XIA	0	231	84	2	160	240	6	10	4	0	1	0	
		YTN	0	452	22	88	160	202	8	13	3	1	1	0	
		HKG	335	361	71	80	120	163	5	8	2	0	0	0	
SHA		0	614	0	87	80	87	5	5	2	0	0	0		
PUS	0	520	25	2	40	44	5	8	4	1	2	1			
R2	C1	OAK	1391	0	72	0	30	0	28	43	10	5	5	3	
		LGB	788	1	211	0	1	0	1	2	1	0	0	0	
		NGB	710	21	71	2	180	0	3	5	1	0	0	0	
		YTN	896	0	133	0	151	0	1	2	0	0	0	0	
		NNS	1331	0	123	34	122	0	10	16	4	2	3	2	
	HKG	696	103	70	123	93	0	10	16	4	1	1	0		
	SHA	760	0	42	25	65	0	29	45	10	5	5	4		
	C2	OAK	626	513	0	149	180	240	0	0	0	0	0	0	
		LGB	0	962	6	125	150	151	7	11	2	1	1	0	
		NGB	0	502	4	90	0	180	16	25	6	4	4	2	
YTN		34	804	27	128	0	152	21	32	8	3	3	2		
NNS		0	801	0	125	0	122	8	12	3	1	0	0		
HKG	0	636	2	124	0	154	0	0	0	0	0	0			
SHA	68	900	0	104	0	95	0	0	0	0	0	0			

Note: R stands for shipping route; C stands for carrier; P stands for port.

Table 6. Slot allocation results for each member at each port (TEU units).

Route	Port	Carrier 1			Carrier 2		
		Laden	Empty	Total TEUs	Laden	Empty	Total TEUs
Route 1	LAX	3544	0	3544	2811	124	2935
	XIA	936	25	961	1190	31	1221
	YTN	1042	5	1047	1666	40	1706
	HKG	1974	10	1984	1734	23	1757
	SHA	1253	6	1259	1656	17	1673
	PUS	959	118	1077	1197	31	1228
Route 2	OAK	1493	145	1638	2610	145	2755
	LGB	1002	6	1008	3902	0	3902
	NGB	1007	14	1021	3902	34	3936
	YTN	1180	5	1185	3902	88	3990
	NNS	1644	57	1701	3902	106	4008
	HKG	1311	49	1360	3902	37	3939
SHA	917	152	1069	3902	0	3902	

business and shipping operations. The Asian ports on the transoceanic cooperative route are typically regional large-scale container ports. Carrier 1 and Carrier 2 use these Asian ports as hubs to efficiently carry out freight collection and distribution. Therefore, based on market demand the two companies arrange different types of laden or empty containers.

6. CONCLUSIONS. The contribution of this study is twofold. First, this study explores synergy management of slot sharing for liner carriers' cooperation through an optimisation approach that accounts for all relevant factors. Second, this research could provide a practical tool for the real-world decision making of slot co-chartering and co-allocation. The analyses indicate the potential use of the methodological framework developed in this paper in the context of cooperative capacity sharing between maritime carriers.

Future effort can be undertaken as follows. First, the relation between successful long-term fleet co-deployment planning and container slot sharing should be considered. Secondly, door-to-door container transport services can be incorporated into shipping alliance cooperative planning. Thirdly, the model should be adapted in response to uncertain transportation demand.

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