



# Developing a dynamic neighborhood structure for an adaptive hybrid simulated annealing – tabu search algorithm to solve the symmetrical traveling salesman problem

Yu Lin<sup>a</sup>, Zheyong Bian<sup>b,\*</sup>, Xiang Liu<sup>b</sup>

<sup>a</sup> College of Management & Economics, Tianjin University, 92 Weijin Road, Nankai District, Tianjin, 300072, China

<sup>b</sup> Department of Civil and Environmental Engineering, Rutgers, The State University of New Jersey, CoRE 736, 96 Frelinghuysen Road, Piscataway, NJ, 08854-8018, United States



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## ABSTRACT

This paper applies a hybrid *simulated annealing – tabu search* algorithm to solve the Traveling Salesman Problem (TSP). Fully considering the characteristics of the hybrid algorithm, we develop a dynamic neighborhood structure for the hybrid algorithm to improve search efficiency by reducing the randomness of the conventional 2-opt neighborhood. A circle-directed mutation is developed to achieve this dynamic neighborhood structure. Furthermore, we propose adaptive parameters that can be automatically adjusted by the algorithm based on context specific examples. This negates the need to frequently readjust algorithm parameters. We employ benchmarks obtained from TSPLIB (a library of sample instances for the TSP) to test our algorithm, and find that the proposed algorithm can obtain satisfactory solutions within a reasonable amount of time. The experimental results demonstrate that the proposed hybrid algorithm can overcome the disadvantages of traditional simulated annealing and tabu search methods. The results also show that the dynamic neighborhood structure is more efficient and accurate than the classical 2-opt. Also, adaptive parameters are appropriate for almost all of the numerical examples tested in this paper. Finally, the experimental results are compared with those of other algorithms, to demonstrate the improved accuracy and efficiency of the proposed algorithm.

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## 1. Introduction

The Traveling Salesman Problem (TSP) is a classical combinatorial optimization problem, originally presented by Dantzig (1959). The TSP belongs to the class of NP-complete problems [1]. In other words, its computational complexity increases exponentially as the number of cities increases. The TSP has received widespread attention in the last 50 years and remains an important research topic. The TSP is often selected to test the performance of newly developed algorithms. Furthermore, the TSP can be assimilated to some other combinatorial optimization problems, such as vehicle routing problems [2–5], pickup and delivery problems [6,7], transportation and logistics problems [8–10], and assembly line sequencing and balancing problems [11].

The concept of TSP can be illustrated as follows. A salesman departs from a city, visits all cities once and only once, and finally returns to the city from which he departed. The objective is to minimize the total distance the salesman must travel to visit all of the cities. We employ the graph theory ( $G = (V, A)$ , where  $V$  represents the set of cities and  $A$  is the set of all the arcs between each set of two cities) to describe the problem. The problem can be formulated as follows [76]:

$$\text{Minimize } z = \sum_{i \in V} \sum_{j \in V} l_{ij} x_{ij} \quad (1)$$

$l_{ij}$  represents the distance between the cities  $i$  and  $j$ .  $x_{ij}$  is the decision variable. If the salesman travels from city  $i$  to city  $j$  directly,  $x_{ij} = 1$ , otherwise,  $x_{ij} = 0$ .

Subject to

$$\sum_{i \in V} x_{ij} = 1, j \in V \quad (2)$$

\* Corresponding author at: Rutgers, The State University of New Jersey, Department of Civil and Environmental Engineering, CoRE 736, 96 Frelinghuysen Road, Piscataway, NJ 08854-8018, United States.

E-mail addresses: [linyu@tju.edu.cn](mailto:linyu@tju.edu.cn) (Y. Lin), [zb91@scarletmail.rutgers.edu](mailto:zb91@scarletmail.rutgers.edu), [zheyongbian@gmail.com](mailto:zheyongbian@gmail.com) (Z. Bian), [xiang.liu@rutgers.edu](mailto:xiang.liu@rutgers.edu) (X. Liu).

$$\sum_{j \in V} x_{ij} = 1, i \in V \quad (3)$$

$\bar{x}$  forms a Hamiltonian cycle, where  $\bar{x}$  represents the vector of decision variables  $x_{ij}$  (4)

$$x_{ij} = \{0, 1\}, i, j \in V. \quad (5)$$

Formula (1) is the objective function that aims to minimize the total length that the salesman must travel. Formulas (2)–(4) ensure that the route forms a legal solution. Formula (5) signifies the range of decision variable  $x_{ij}$ .

In this paper, only symmetrical traveling salesman problems are considered, which indicates that  $l_{ij} = l_{ji}$ . In fact,  $l_{ij}$  and  $l_{ji}$  are identical elements that belong to the arc set  $A$ .

Many efficient heuristic algorithms have been developed to solve the TSP. Among these algorithms, simulated annealing and tabu search algorithms are among the simplest yet effective methods. They are widely applied in many different research fields, such as vehicle routing [2], scheduling [52–55], transportation and logistics [8,10], assembly sequencing [55–57], facilities location [58], etc. The effectiveness of simulated annealing and tabu search to solve the TSP has been verified by numerous studies [19–23,59–65]. However, both simulated annealing and tabu search have some disadvantages. Simulated annealing requires a sufficient number of iterations to guarantee a high-quality solution and is sensitive to parameter selection. In addition, repeated searches are inevitable because the search may return to old solutions and oscillate between solutions without the memory function. In contrast, tabu search algorithm has a fast decrease rate and can prevent repetitive searches by using a tabu list function. However, it cannot guarantee convergence. Its performance largely depends on the initial solution. Moreover, owing to lack of effective control, the later stages of the tabu search process seem to move randomly at a comparatively low objective function value. Therefore, we incorporate the Metropolis acceptance criteria [66] of simulated annealing into tabu search to control the movement during the search process. A hybrid simulated annealing – tabu search algorithm is expected to eliminate some of the main weaknesses of simulated annealing and tabu search.

In addition, the effectiveness of both tabu search and simulated annealing algorithms largely depends on the neighborhood structure. The most commonly used method is the classical  $\lambda$ -opt neighborhood structure [67,68]. However,  $\lambda$ -opt has a random component. In other words, during iterative process of an algorithm, the  $\lambda$ -opt randomly generates neighborhood solutions. We fully consider the properties of the hybrid algorithm and propose an effective neighborhood structure, called dynamic neighborhood structure. A circle-directed mutation is developed to achieve the dynamic neighborhood structure. This structure is more adaptive and can eliminate a certain amount of the randomness of traditional neighborhoods. In other words, low-quality mutation can be avoided through the dynamic neighborhood. A detailed description of the structure will be provided in Section 3.1. The experimental results demonstrate that this dynamic neighborhood structure can significantly reduce computational time and improve the quality of solutions obtained.

Finally, since the simulated annealing algorithm is sensitive to parameters, and each iteration of our algorithm is conducted by simulated annealing, the setting of parameters affects the performance of the hybrid algorithm. Thus, this paper proposes a set of reasonable parameters. We first analyze the influence of some characteristics of a specific benchmark on the setting of parameters and subsequently propose a set of adaptive parameters that can be auto-

**Table 1**  
Heuristic methods applied to TSP.

References	LK	SLS	SA	TS	GA	ACO	PSO	HBMO	NN
[12–17,71–73]	✓								
[12,15,68]		✓							
[18–21,59–62]			✓						
[22,23,61]				✓					
[25–27,29,30]					✓				
[32–37,78]						✓			
[38–41,79]							✓		
[42,43]								✓	
[44–47,74]									✓
[22]	✓			✓					
[31]		✓			✓				
[24,28]	✓				✓				
[48]		✓				✓			
[50,51]					✓		✓		
[49]			✓		✓		✓		
[75]	✓					✓	✓		
[77]						✓			✓

Notes:

LK: Lin-Kernighan implementations and Lin-Kernighan-based heuristics.

SLS: Simple local search algorithms.

SA: Simulated annealing algorithms.

TS: Tabu search algorithms.

GA: Genetic algorithms.

ACO: Ant colony optimization algorithms.

PSO: Particle swarm optimization algorithms.

HBMO: Honey bee mating optimization algorithms.

NN: Neural networks.

matically adjusted by the algorithms based on these characteristics. The computational complexity of the developed algorithms is only  $O(N^{1.1})$  based on the proposed adaptive parameters, where  $N$  is the number of cities representing the scale of a problem.

This paper is structured as follows: Section 2 provides a literature review on recent related works. Section 3 introduces the adaptive hybrid simulated annealing – tabu search algorithm with a dynamic neighborhood structure. The computational experiments and results are presented in Section 4. Finally, conclusions are provided in Section 5 and related future work is described in Section 6.

## 2. Literature review

### 2.1. Existing algorithms

In the past two decades, many efficient heuristic algorithms have been developed to solve the TSP. These include simple local search algorithms (such as 2-opt and 3-opt local search algorithms), Lin-Kernighan (LK) implementations and Lin-Kernighan-based heuristics, simulated annealing (SA), tabu search (TS), genetic algorithms (GA), ant colony optimizations (ACO), particle swarm optimizations (PSO), newly developed honey bee mating optimizations (HBMO), neural networks (NN), and some hybrid algorithms. Table 1 summarizes the heuristic algorithms applied to solve the TSP.

In recent years, a number of studies are focusing on the improvement of traditional heuristics, mainly through two methods: (1) developing hybrid algorithms to overcome the disadvantages of individual algorithms, and (2) developing effective mutation methods for the iterative process.

Hybrid algorithms can combine the advantages and overcome the disadvantages of individual algorithms. Thus, hybrid algorithms are usually more effective than each individual algorithm. Nguyen et al. [24] and Baraglia et al. [28] developed a hybrid algorithm combining the genetic algorithm (GA) with the Lin-Kernighan heuristic for the TSP and successfully found high-quality solutions. Wang [31] proposed two novel local searches and incorporated them

into a genetic algorithm. The hybrid genetic algorithm found better approximate solutions than the traditional genetic algorithms. Tsai et al. [48] introduced the multiple ant colonies concept from parallel genetic algorithms to search for solutions and proposed two novel local searches for their algorithms. Chen & Chien [49] combined the genetic algorithm, simulated annealing, ant colony system, and particle swarm optimization to solve the TSP and obtained satisfactory solutions. Chen & Chien [50] and Dong et al. [51] developed hybrid algorithms combining the genetic algorithm with ant colony optimization for the TSP. Both of them verified the effectiveness of the proposed algorithms. Mahi et al. [75] employed a hybrid method based on particle swarm optimization, ant colony optimization, and 3-opt algorithms to solve the TSP. They demonstrated that the algorithm can overcome the disadvantages of individual algorithms used.

On the other hand, the efficiency and accuracy of heuristic algorithms largely depend on mutation methods. Mutation represents the change in solutions between two iterations, such as the neighborhood structure of local-search-based algorithms, crossover in genetic algorithms, mutation defined by the PSO formula in particle swarm optimization, etc. The Lin-Kernighan heuristic [13] proposed in 1973 is one of the most effective mutation methods for local-search-based algorithms. Geng [21] incorporated three neighborhood structures into a simulated annealing algorithm to improve search efficiency. For some swarm algorithms, several researchers also proposed improvements for mutation methods. For example, Nagata and Kobayashi [30] proposed edge assembly crossover for genetic algorithms. The experiment resulted in high-quality solutions obtained by the designed algorithm. Marinakis et al. [43] designed a multiple phase neighborhood search-greedy randomized adaptive search procedure (MPNS-GRASP), expanding neighborhood search (ENS), and a new crossover operator based on an adaptive memory procedure for the honey bees mating optimization algorithm to solve the TSP with high efficiency and accuracy. Both Albayrak & Allahverdi [29] and Wang [31] developed new local search mutations for genetic algorithms. Escario et al. [37] developed two new search strategies for ant colony optimization (ACO) and formed the ant colony extended (ACE) algorithm. Their experiment showed the ACE's superiority over the traditional ACO.

## 2.2. Knowledge gaps

Lin-Kernighan implementations and Lin-Kernighan-based heuristics (LK) and simple local search (SLS) algorithms have strong ability of local search but lack global search ability. In contrast, swarm algorithms, genetic algorithms (GA), ant colony optimization algorithms (ACO) and honey bee mating optimization algorithms (HBMO) have strong ability of global search, but have relatively low ability of local search. Simulated annealing and tabu search have both strong local search ability and global search ability. The hybrid simulated annealing – tabu search algorithm's advantages can overcome each individual algorithms' disadvantages. In fact, designing hybrid algorithms and advanced mutation methods is an emerging research subject for solving combinatorial optimization problems. However, very little prior research has demonstrated the effectiveness of a hybrid simulated annealing – tabu search algorithm to solve the TSP. In addition, most of the advanced mutation methods in the literature are static, except a few variable neighborhood structures like Lin-Kernighan neighborhoods. Although these variable neighborhoods are dynamic, they still exhibit considerable randomness in neighbor solution selection during iterative process and are not adaptive to different iterative stages. Finally, parameters have an influence on the performance of an algorithm, yet this influence has received relatively little attention. Geng et al. [21] alone has proposed a set of adaptive parameters for simulated annealing

using a greedy search, but their parameter setting techniques are not entirely applicable to our algorithms. Moreover, they did not study the relationship between the characteristics of numerical examples and the setting of parameters. All these deficiencies in the current work incentivized this paper to propose a hybrid simulated annealing – tabu search algorithm based on dynamic neighborhood structure with adaptive parameters for the TSP.

## 2.3. Intended contributions of this paper

To narrow the above-mentioned knowledge gaps, this paper aims to bring the following contributions to the body of knowledge:

- This paper represents one of the earliest studies that apply hybrid simulated annealing – tabu search algorithm to solve the TSP.
- This paper proposes the concept of dynamic neighborhood structure, which aims to promote the algorithms' adaptiveness to different iterative stages and reduce randomness of traditional mutation methods. This is an improvement to previous neighborhood search methods.
- This paper studies the relationship between the characteristics of context specific examples and the parameters used in the algorithm. Adaptive parameters are determined based on characteristics of specific examples.

## 3. The adaptive hybrid simulated annealing – tabu search algorithm with a dynamic neighborhood structure based on a circle-directed mutation (AHSATS-D-CM)

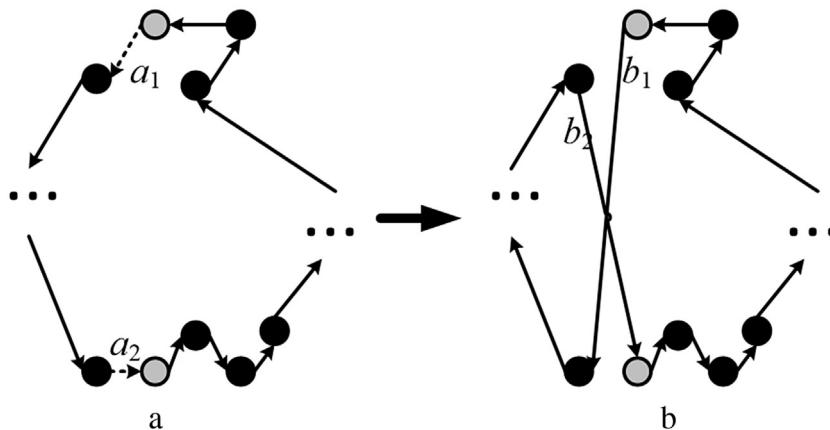
In this section, we review the classical neighborhood 2-opt and provide a detailed introduction for the dynamic neighborhood structure based on a circle-directed mutation. Then two hybrid simulated annealing – tabu search algorithms are introduced in Section 3.2, one based on classical 2-opt (AHSATS-2-opt) and the other based on the dynamic neighborhood structure using the circle-directed mutation (AHSATS-D-CM). Finally, in Section 3.3, we analyze several characteristics of numerical examples and propose the adaptive parameters for the two hybrid algorithms according to these characteristics.

### 3.1. The dynamic neighborhood structure based on the circle-directed mutation (D-CM)

A dynamic neighborhood structure is defined as the neighbor solutions that are dynamically selected and are adaptive to different stages of an algorithm. Therefore, the algorithm can reduce randomness of neighbor solution generations and avoid low-quality mutations. In this paper, the dynamic neighborhood structure is designed for the classical 2-opt neighborhood structure.

Two-opt is one of the simplest neighborhood structures, but it is effective for simulated annealing and tabu search [21,22]. It can be described as follows: two links in the current tour are replaced by two new links. In other words, the new tour is obtained by deleting two links and joining the resulting paths together in a new arrangement.

Most meta-heuristic algorithms have the iterative process in which low-quality solutions are generated at the initial stage, with the quality of solutions improving as the number of iterations increases, and finally high-quality solutions are obtained at the later stage. Our hybrid algorithm is no exception. The common classical 2-opt neighborhood structure has a randomized component, in which the two edges are randomly selected to be replaced by two new edges. The random 2-opt can perform global searches at the initial stage of an algorithm when the objective function value of solutions reached is relatively high. However, owing to

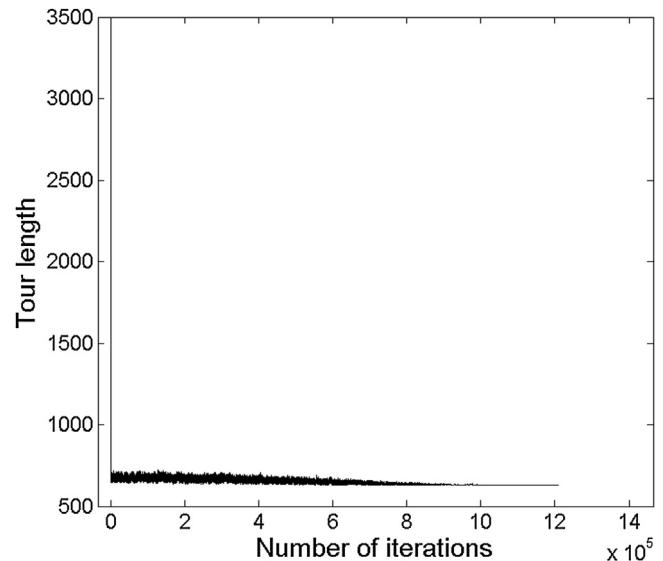


**Fig. 1.** A random mutation of 2-opt based on a short tour.

its randomness, it becomes inefficient when better-quality solutions are obtained at the later stage of an algorithm. It will expend unnecessary search time and even break the high-quality solutions obtained at the later stage of an algorithm. This can be demonstrated through the following example. Fig. 1 (a) represents a short tour obtained by an algorithm during a later stage of execution. At this moment, the algorithm generates a random mutation of 2-opt, as Fig. 1 (b) shows, with a significant increase in the total length, because  $b_1 + b_2 - a_1 - a_2$  is a number much larger than zero. Although such mutations may be generated as solution candidates, it is unlikely to be selected by the tabu search. Thus, such random mutations will expend unnecessary search time during the later stage of the algorithm. Fig. 2 demonstrates that the hybrid algorithm proposed in this paper has a very fast decrease rate. This indicates that short tours are obtained in few iterations. The algorithm will be inefficient and perform many unnecessary iterations if the random 2-opt neighborhood structure is adopted for the overall search process.

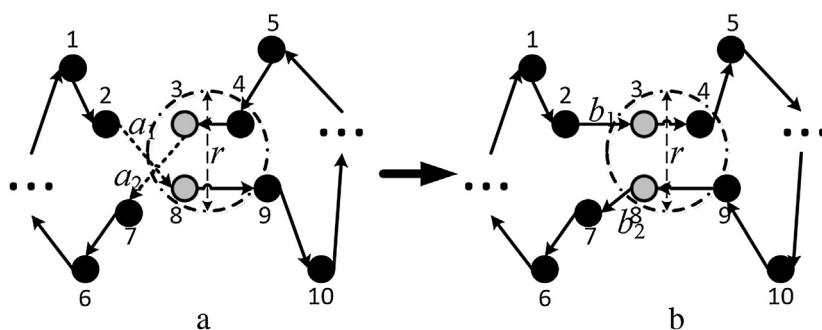
To overcome the disadvantage of random searches during the search stage, we develop a dynamic neighborhood structure using circle-directed mutation in place of the random mutation.

Fig. 3 (a) represents a short tour obtained during the later stage of execution. We first find a random location in the graph  $G = (V, A)$ , where  $V$  is the set of all vertexes (i.e. cities) and  $A$  is the set of all arcs connecting any two vertexes. We then draw a circle with a diameter of  $r$ , which is longer than  $p$  percent of the arcs ( $A$ ) in graph  $G$ , and then reverse the visiting order of the cities between two randomly selected vertexes (cities) within the circle. Here, we define the diameter of the circles by determining the percentage  $p$ . First, we sequence all arcs ( $A$ ) in the graph from the shortest to the longest; we then obtain the percentage  $p$  of the shortest arcs (the

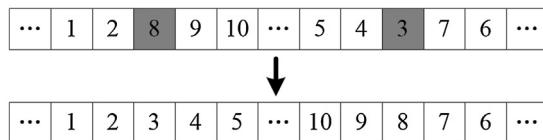


**Fig. 2.** Iterative process of the hybrid simulated annealing – tabu search algorithm.

set is defined as  $PA$ ) that accounts for the arcs in set  $A$ . The longest arc in  $PA$  is the circle's diameter. We take Fig. 3 as an example. Given a specific value of percentage  $p$ , the diameter of the circles  $r$  is thus determined. Hypothetically, the set  $PA$  contains these arcs: (1, 2), (2, 3), (2, 7), (2, 8), (3, 4), (3, 7), (3, 8), (3, 9), (4, 5), (4, 8), (4, 9), (6, 7), (7, 8), (8, 9), (9, 10), etc. whose lengths are all no greater than the diameter  $r$ . We randomly select a pair of vertexes from  $PA$ ; in this example, the pair (3, 8) is selected. Then, we reverse



**Fig. 3.** A circle-directed mutation of 2-opt based on a short tour.



**Fig. 4.** An example of mutation.

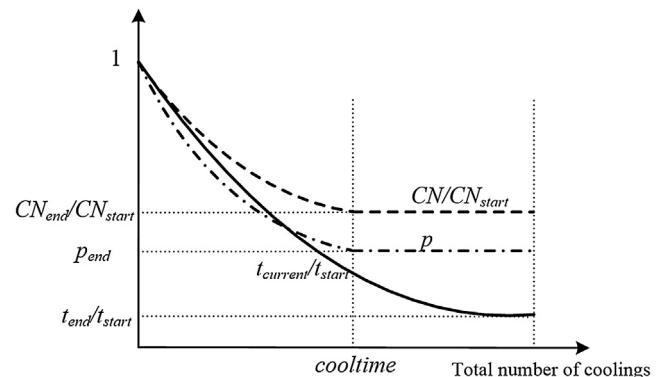
the visiting order of the cities between nodes 3 and 8, as Fig. 4 shows. Poor-quality mutations, such as a reversal of the traversal sequence between vertexes 1 and 10, are excluded by the circle restriction. This circle-directed mutation decreases the probability of poor-quality mutations during the later stage of an algorithm because the circle restricts the length of newly generated edges ( $b_1$  and  $b_2$  in Fig. 3).

The percentage  $p$  of the shortest arcs that accounts for the arcs in set  $A$  is not a constant. At the initial stage of the hybrid algorithm, the tour lengths reached are relatively long. Therefore, the circles should be large enough to ensure that all pairs of vertexes have a probability of being selected to achieve the mutation so that the algorithm has global search capability. In contrast, when shorter tours are generated in the later stage of the algorithm, the diameter of circles should be decreased to rule out poor-quality mutations. Therefore, the mechanism of the hybrid algorithm requires that the percentage  $p$  should begin at a high ratio and end at a relatively low value. The objective function value (the tour length) decreases very fast at the initial stage of the hybrid algorithm, while its decrease rate slows down during the late stage of the search process (as Fig. 2 shows). The decrease rate of the percentage  $p$  should correspond to that of the objective function value. We anticipate that it is appropriate for the percentage  $p$  to decrease at an exponential rate whose decrease rate is high at the initial stage while slowing at the later stage. The dynamic neighborhood structure is thus proposed based on considerations above, and the detailed settings of the structure are described in Sections 3.2 and 3.3.

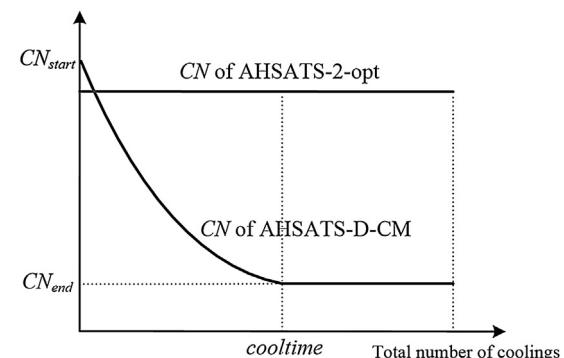
### 3.2. The detailed mechanism of AHSATS-2-opt and AHSATS-D-CM

For AHSATS-2-opt, simulated annealing begins at the initial temperature ( $t_{start}$ ), and ends at the final temperature ( $t_{end}$ ). The temperature is decreased at an exponential rate via multiplication by a cooling coefficient ( $t_{cool}$ ) which is slightly lower than 1. At each temperature, SA performs  $Elen$  (epoch length) iterations and accepts a newly generated solution ( $X_{opt}$ ) with a probability of  $\rho$  at each iteration. The tabu search is responsible for generating a new solution ( $X_{opt}$ ) based on a current solution ( $X_{current}$ ). We employ the 2-opt neighborhood to generate  $CN$  candidate solutions. Among the candidate solutions, the solution with the shortest tour length that conforms to the aspiration criterion or is not in the tabu list is selected as  $X_{opt}$  – the solution waits to be accepted (or not) by the Metropolis acceptance criteria of simulated annealing. The structure of AHSATS2-opt is listed in pseudo-code in Table 2.

For AHSATS-2-opt, the operational rules of SA are the same as those of AHSATS-2-opt. We employ the dynamic neighborhood structure proposed in Section 2.1, instead of the classical 2-opt usually used in tabu search. In the initial stage of the algorithm, the objective function value is high (as Fig. 2 shows); thus the percentage  $p$  must be initiated at a high value ( $p_{start}$ ) to ensure a globally scoped search; simultaneously, the initial number of candidate solutions ( $CN_{start}$ ) generated by the tabu search is comparatively large. However, the objective function value decreases rapidly during the search process. Therefore, the percentage  $p$  and the number of candidate solutions  $CN$  should be decreased to avoid unnecessary calculations. We stipulate that the percentage  $p$  and the number of candidate solutions ( $CN$ ) decrease as the temperature decreases



**Fig. 5.** Dynamic parameters of AHSATS-D-CM.



**Fig. 6.** Comparison of the number of candidate solutions generated between AHSATS-2-opt and AHSATS-D-CM.

every time, as Fig. 5 shows. The two values also decrease at an exponential rate via multiplication by the decrease coefficients  $p_{cool}$  and  $CN_{cool}$ , respectively. Both of them stop decreasing when  $p_{end}$  and  $CN_{end}$  are reached. This decrease is performed  $cooltime$  times. The comparison between the  $CN$ s of AHSATS-2-opt and AHSATS-D-CM during the search process is shown in Fig. 6. We can conclude from Fig. 6 that, compared with AHSATS-2-opt, AHSATS-D-CM generates fewer neighbor solutions and requires fewer calculations of tour length differences.

The structure of AHSATS-D-CM is given in Table 3 and Fig. 7.

### 3.3. The adaptive parameters

For almost all algorithms, parameters must be adjusted to adapt to different numerical examples. It is time-consuming to adjust the parameters for different numerical examples every time an algorithm is executed, especially for algorithms in which parameters have a significant influence on the solution quality. We focus on proposing a set of adaptive parameters that can be adjusted automatically according to the characteristics of a specific numerical example. We have analyzed several characteristics of the benchmarks in TSPLIB and their influence on the parameters in our algorithms. Based on this analysis, a set of adaptive parameters is determined by testing benchmarks from TSPLIB.

#### 3.3.1. Four scalars representing the characteristics of numerical examples

Before the adaptive parameters are proposed, we first define four scalars that can reflect several characteristics of the benchmarks. These scalars will be used in the following parameter settings.

**Table 2**

Algorithm 1: AHSATS-2-opt.

```

Input the data of a numerical example
Initialize the parameters of the algorithm:  $t_{start}$ ,  $t_{end}$ ,  $t_{cool}$ ,  $CN$ ,  $TL$  (length of
tabu list),  $Elen$  and set  $t_{current} = t_{start}$ ,  $it = 0$ ,  $X_{current} = X_0$ ,  $E_{current} = f(X_0)$ ,  $X_{best} =$ 
 $X_0$ ,  $E_{best} = f(X_0)$ 
Do while  $t_{current} > t_{end}$ 
  Do while  $it < Elen$ 
    Use the neighbor structure (2-opt) to generate  $CN$  candidate
    solutions  $\{X_1, X_2, \dots, X_{CN}\}$  of  $X_{current}$ 's neighbors.
    Calculate  $\{\Delta f_1, \Delta f_2, \dots, \Delta f_{CN}\}$  ( $\Delta f_i = f(X_i) - E_{current}$ ) and record the
    subscript  $opt$ , where  $\Delta f_{opt} = \min\{\Delta f_1, \Delta f_2, \dots, \Delta f_{CN}\}$ .
    If  $E_{current} + \Delta f_{opt} < E_{best}$  (the aspiration criterion)
       $X_{current} = X_{opt}$ 
       $E_{current} = E_{current} + \Delta f_{opt}$ 
       $X_{best} = X_{opt}$ 
       $E_{best} = E_{current} + \Delta f_{opt}$ 
      Update the tabu list
    Else
      Do while the mutation from  $X_{current}$  to  $X_{opt}$  is in tabu list
        Select the suboptimal solution as  $X_{opt}$  from  $\{X_1,$ 
         $X_2, \dots, X_{CN}\}$ 
      End do
      If  $\Delta f_{opt} < 0 \parallel \rho < rand$  (see formula 10)
         $X_{current} = X_{opt}$ 
         $E_{current} = E_{current} + \Delta f_{opt}$ 
        Update the tabu list
      End if
    End if
     $it = it + 1$ 
  End do
   $t_{current} = t_{current} \times t_{cool}$ 
End do
Output  $X_{best}$  and  $E_{best}$ .

```

1)  $N$ : number of cities.

The number of cities reflects the scale of a numerical example. The parameters in most algorithms depend on the scale of numerical examples, and our algorithms are no exception.

$$2) \alpha = \frac{|a|}{A}, a = \left\{ l \in A \mid |l| \leq \max_{j} \min_{i} (|l_{ij}|), l_{ij} \in A; i, j \in V \right\} \quad (6)$$

If the variable between the vertical bars ( $| |$ ) represents a set (such as  $a$  and  $A$ ), it receives the number of elements in the set; if the variable between the bars represents an arc (such as  $l$ ), it receives the length of the arc.

The scalar  $\alpha$  is used in the dynamic neighborhood structure. It determines the value of  $p_{end}$ , which must be large enough (compared to the ratio  $\alpha$ ) to ensure that every vertex (i.e. city) has a sufficient probability of being selected to generate neighbor solutions.

$$3) \beta = \sum_{l \in K} |l|, K = \left\{ l \mid |l| = \min_i (|l_{ij}|), l_{ij} \in A; i, j \in V \right\} \quad (7)$$

where  $|l|$  is the length of arc  $l$ .

The scalar  $\beta$ , which is the sum of the lengths of the shortest arcs linking all the vertexes, is used to determine the acceptance probability  $\rho$  in simulated annealing. In general, the initial temperature  $t_{start}$  and the final temperature  $t_{end}$  in simulated annealing

**Table 3**

Algorithm 2: AHSATS-D-CM.

```

Input the data of a numerical example
Initialize the parameters of the algorithm:  $t_{start}$ ,  $t_{end}$ ,  $t_{cool}$ ,  $CN_{start}$ ,  $TL$ ,  $CN_{cool}$ ,  

 $p_{cool}$ ,  $Elen$  and set  $p=1$ ,  $CN=CN_{start}$ ,  $t_{current}=t_{start}$ ,  $it=0$ ,  $X_{current}=X_0$ ,  $E_{current}=\mathcal{f}(X_0)$ ,  $X_{best}=X_0$ ,  $E_{best}=\mathcal{f}(X_0)$ 
Do while  $t_{current} > t_{end}$ 
  Do while  $it < Elen$ 
    Use the dynamic neighborhood structure to generate  $CN$   

    candidate solutions  $\{X_1, X_2, \dots, X_{CN}\}$  of  $X_{current}$ 's neighbors.  

    Calculate  $\{\Delta f_1, \Delta f_2, \dots, \Delta f_{CN}\}$  ( $\Delta f_i = \mathcal{f}(X_i) - E_{current}$ ) and record the  

    subscript  $opt$ , where  $\Delta f_{opt} = \min\{\Delta f_1, \Delta f_2, \dots, \Delta f_{CN}\}$ .
    If  $E_{current} + \Delta f_{opt} < E_{best}$ 
       $X_{current} = X_{opt}$ 
       $E_{current} = E_{current} + \Delta f_{opt}$ 
       $X_{best} = X_{opt}$ 
       $E_{best} = E_{current} + \Delta f_{opt}$ 
      Update the tabu list
    Else
      Do while the mutation from  $X_{current}$  to  $X_{opt}$  is in tabu list
        Select the suboptimal solution as  $X_{opt}$  from  

         $\{X_1, X_2, \dots, X_{CN}\}$ 
      End do
      If  $\Delta f_{opt} < 0 \parallel \rho < rand$  (see formula 10)
         $X_{current} = X_{opt}$ 
         $E_{current} = E_{current} + \Delta f_{opt}$ 
        Update the tabu list
      End if
    End if
     $it = it + 1$ 
  End do
   $t_{current} = t_{current} \times t_{cool}$ 
   $CN = \max(CN \times CN_{cool}, CN_{end})$ 
   $p = \max(p \times p_{cool}, p_{end})$ 
End do
Output  $X_{best}$  and  $E_{best}$ .

```

should be adjusted according to a specific numerical example, and the acceptance probability only depends on the current temperature. However, in our algorithms,  $t_{start}$  and  $t_{end}$  are stipulated to be constants for convenience; thus we must adjust the acceptance probability  $\rho$  according to different numerical examples.

$$4) \gamma = \frac{N \cdot std(|l|)}{\sum_{l \in K} |l|}, K = \left\{ l \mid l_i = \min_i (|l_{ij}|), l_{ij} \in A; i, j \in V \right\} \quad (8)$$

where  $std()$  calculates the standard deviation.

The scalar  $\gamma$  can reflect the degree of irregularity and dispersion of a specific numerical example. The degree of irregularity and dispersion of the numerical example increases with the scalar  $\gamma$  increasing. The degree of irregularity and dispersion has a significant influence on the following parameters: number of candidate solutions  $CN$  (including  $CN_{start}$  and  $CN_{end}$ ), epoch length  $Elen$ , and acceptance probability  $\rho$ .

We apply benchmarks pr226 and ts225 from TSPLIB to demonstrate the relationship between the parameters ( $CN$  and  $Elen$ ) and the degree of irregularity and dispersion. Figs. 8 and 9 show distribution diagrams for pr226 and ts225, respectively. It is evident

**Table 4**

Experimental results of ts225 and pr226 under different settings of parameters (*CN* and *Elen*).

Instances	Error <sub>avg</sub>				
	<i>CN</i> = 53 <i>Elen</i> = 20746	<i>CN</i> = 105 <i>Elen</i> = 10373	<i>CN</i> = 210 <i>Elen</i> = 5187	<i>CN</i> = 420 <i>Elen</i> = 2593	<i>CN</i> = 840 <i>Elen</i> = 1297
ts225 ( $\gamma = 0$ )	0.416	0.357	0.233	<b>0.124</b>	0.126
pr226 ( $\gamma = 1.42$ )	0.747	<b>0.415</b>	1.259	1.214	2.280

**Table 5**

Setting of parameters of AHSATS-2-opt.

notations	definition	value
<i>t<sub>start</sub></i>	The initial temperature	50
<i>t<sub>cool</sub></i>	The cooling coefficient of temperature	0.99
<i>t<sub>end</sub></i>	The final temperature	0.15
<i>Elen</i>	Epoch length	Formula (9)
$\rho$	The probability of accepting a newly generated solution	Formula (10)
<i>CN</i>	The number of candidate solutions	Formula (11)
<i>TL</i>	The length of tabu list	Formula (12)

that pr226 has a high degree of irregularity and dispersion, whereas ts225 has a significantly lower degree of irregularity and dispersion. We employ AHSATS-2-opt to conduct an experiment showing the relationship between the numerical example's degree of irregularity and dispersion and the setting of parameters *CN* and *Elen*. We generate five pairs of *CN* and *Elen* parameters. Using each pair of parameters, we conduct six repeated experiments. The experimental results are listed in Table 4, where error<sub>avg</sub> (i.e. (mean length – optimum) × 100/optimum) denotes the percentage gap between the average tour length and the optimal tour length reported in TSPLIB.

The benchmarks ts225 and pr226 have almost the same scale, but the experiment shows that the two benchmarks have significant differences that result from the settings of parameters *CN* and *Elen*: pr226 produces the minimum average error when *Elen* = 10373 and *CN* = 105, while *Elen* = 2593 and *CN* = 420 are optimal for ts225 (the bold values shown in Table 4). The optimal parameters for the two benchmarks have a gap of four times. Thus, we can conclude that the degree of irregularity and dispersion can significantly influence the setting of parameters *CN* and *Elen*.

### 3.3.2. Parameters of AHSATS-2-opt

The initial temperature (*t<sub>start</sub>*), final temperature (*t<sub>end</sub>*), and cooling coefficient (*t<sub>cool</sub>*) are all defined as constants for all numerical examples. In contrast, the SA epoch length (*Elen*), the number of candidate solutions (*CN*), the length of tabu list (*TL*), and the acceptance probability ( $\rho$ ) of a newly generated solution ( $X_{opt}$ ), which is generated by a tabu search, depend on the data of a specific numerical example. The parameter settings are listed in Table 5.

$$Elen = \left[ \frac{5600N^{0.4}(1.27 + \gamma^{4.11})[4.72 \times 10^{-11}(\gamma + 0.1) + N^{-2.81}]}{(22.10 + \gamma^{4.11})(1.42 \times 10^{-11} + N^{-2.81})} \right] \quad (9)$$

where the square brackets “[]” receive the rounding integer.

$$\rho = \begin{cases} \frac{-2.46 \cdot N \cdot [f(X_{opt}) - f(X_{current})]}{t_{current} \cdot \beta \cdot (3.7 + \gamma^{1.1})}, & f(X_{opt}) - f(X_{current}) > 0 \\ 1, & f(X_{opt}) - f(X_{current}) \leq 0 \end{cases} \quad (10)$$

where  $X_{opt}$  is newly generated by the tabu search,  $X_{current}$  is the current solution,  $t_{current}$  is the current temperature, and the function “f()” calculates the tour length.

**Table 6**

Setting of parameters of AHSATS-D-CM.

notations	Definition	value
<i>t<sub>start</sub></i>	The initial temperature	40
<i>t<sub>cool</sub></i>	The cooling coefficient of temperature	0.99
<i>t<sub>end</sub></i>	The final temperature	0.15
<i>Elen</i>	Epoch length	Formula (13)
$\rho$	The probability of accepting a newly generated solution	Formula (10)
<i>cooltime</i>	The number of decreases of <i>p</i> and <i>CN</i>	Formula (14)
<i>p</i>	The percentage of PA that accounts for <i>A</i>	–
<i>p<sub>start</sub></i>	Initial value of percentage <i>p</i>	1
<i>p<sub>end</sub></i>	Final value of percentage <i>p</i>	Formula (15)
<i>p<sub>cool</sub></i>	Decrease coefficient of <i>p</i>	Formula (16)
<i>CN</i>	The number of candidate solutions	–
<i>CN<sub>start</sub></i>	Initial number of candidate solutions	Formula (17)
<i>CN<sub>end</sub></i>	Final number of candidate solutions	Formula (18)
<i>CN<sub>cool</sub></i>	Decrease coefficient of <i>CN</i>	Formula (19)
<i>TL</i>	The length of tabu list	Formula (20)

In [21], the authors have already defined the parameter  $\rho$  similarly:

$$\rho = e^{-[F(c^{best}) - F(c^{old})]/t_{current}} \times (10 \times N/OPT).$$

However, the optimal tour length (*OPT*) is used in their formula. In typical instances, the optimal tour length is unknown, and thus their formula had limitations. In this paper, we modify the formula in Ref. [21], and employ the scalars  $\beta$  and  $\gamma$  instead, which can be easily calculated using the data from a specific numerical example.

$$CN = \left[ \frac{2800N^{1.1}}{Elen} \right] \quad (11)$$

$$TL = \left[ \frac{Elen^{0.6}}{3.5} \right] \quad (12)$$

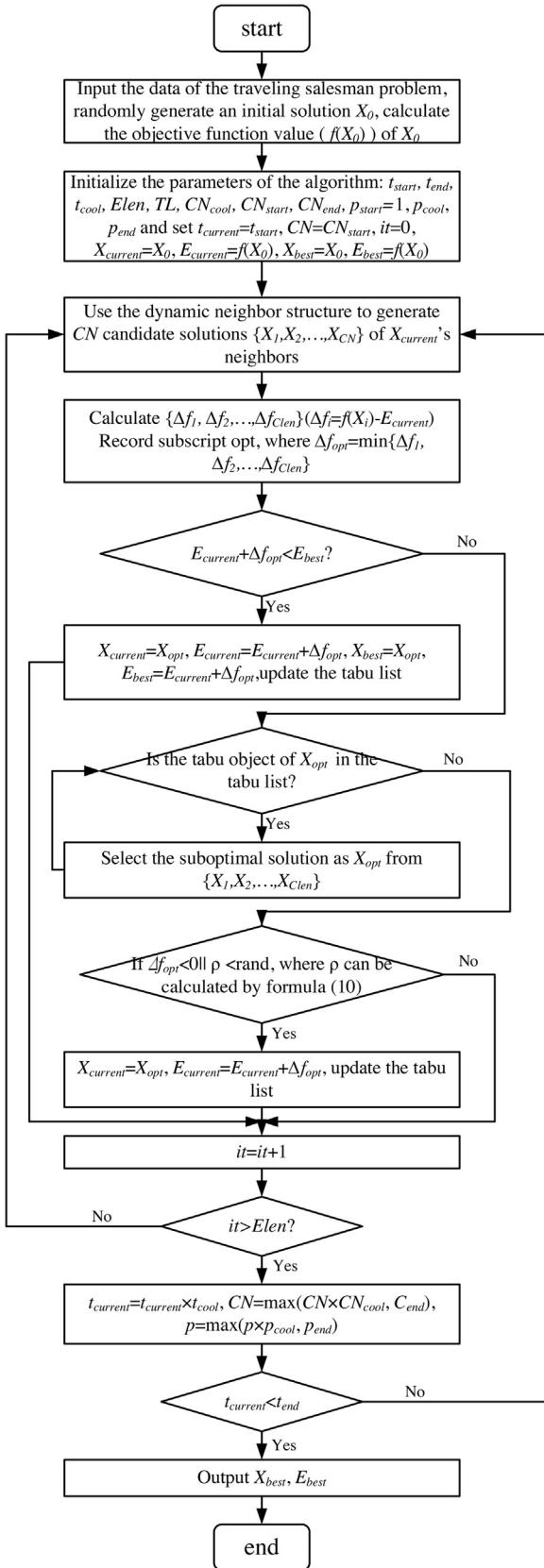
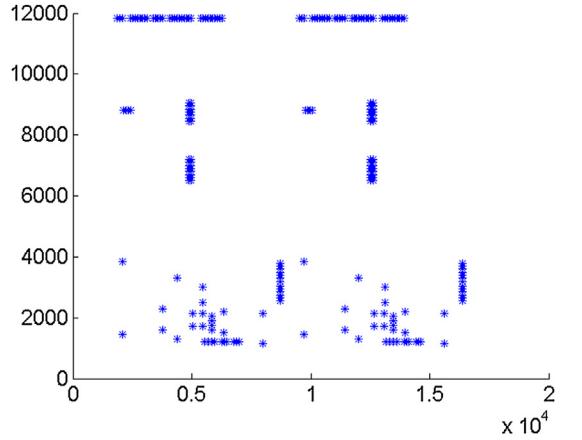
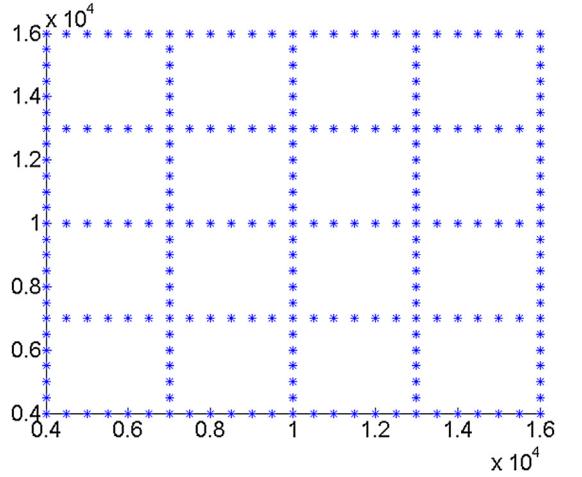
### 3.3.3. Parameters of AHSATS-D-CM

In addition to the parameters defined in AHSATS-2-opt, extra parameters must be set for AHSATS-D-CM: the number of decreases for *p* and *CN* (*cooltime*), initial value of percentage *p* (*p<sub>start</sub>*), final value of percentage *p* (*p<sub>end</sub>*), decrease coefficient of *p* (*p<sub>cool</sub>*), initial number of solution candidates (*CN<sub>start</sub>*), final number of solution candidates (*CN<sub>end</sub>*), and the decrease coefficient of *CN* (*CN<sub>cool</sub>*). The parameter settings are listed in Table 6.

$$Elen = \left[ \frac{5556N^{0.4}(1.28 + \gamma^{4.11})[4.72 \times 10^{-11}(\gamma + 0.1) + N^{-2.81}]}{(24.72 + \gamma^{4.11})(1.42 \times 10^{-11} + N^{-2.81})} \right] \quad (13)$$

$$cooltime = \left[ \frac{(121950 + 2.75N^{2.18}) \cdot \log_{t_{cool}}^{t_{end}/t_{start}}}{121950 + N^{2.18}} \right] \quad (14)$$

$$p_{end} = \min[\max(3 \cdot \alpha, 0.1), 1] \quad (15)$$

**Fig. 7.** Flow chart for AHSATS-D-CM.**Fig. 8.** Distribution diagram for pr226.**Fig. 9.** Distribution diagram for ts225.

$p_{end}$  is defined in Formula (15) to ensure that every vertex (i.e. city) has a sufficient probability of being selected to generate neighbor solutions, so that the algorithm has global search capacity.

$$p_{cool} = p_{end}^{\frac{1}{cooltime}} \quad (16)$$

$$CN_{start} = \left[ \frac{2500N^{1.1}}{Elen} \right] \quad (17)$$

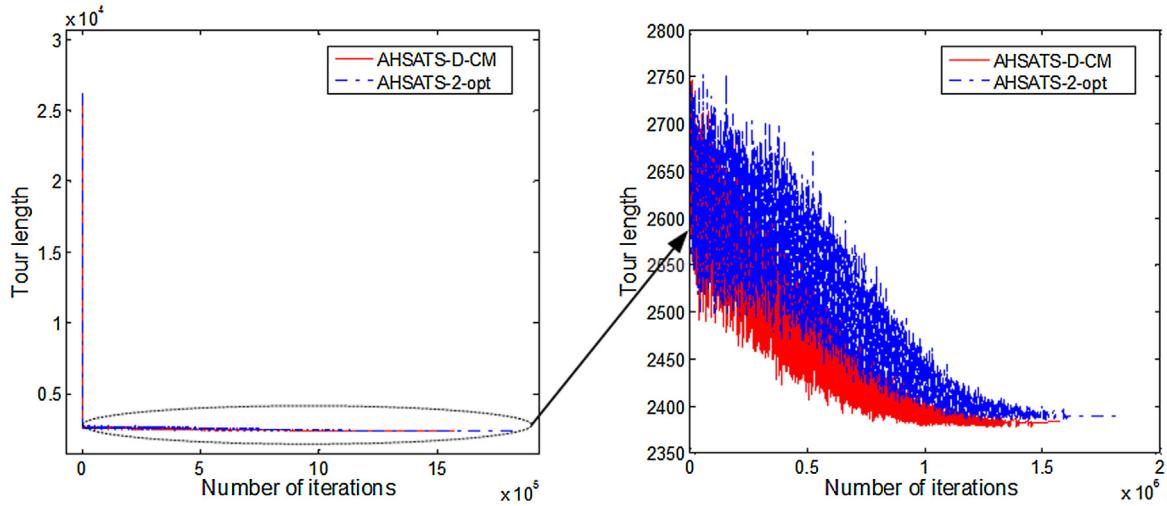
$$CN_{end} = \left[ CN_{start} \frac{1538 + 1.35N^{1.14}}{1538 + N^{1.14}} + p_{end} - 1 \right] \quad (18)$$

$$CN_{cool} = \left( \frac{CN_{start}}{CN_{end}} \right)^{\frac{1}{cooltime}} \quad (19)$$

$$TL = \left[ \frac{Elen^{0.6}}{3.5} \right] \quad (20)$$

### 3.3.4. Analysis of time complexity for AHSATS-2-opt and AHSATS-D-CM

We analyze the time complexity of the two algorithms AHSATS-2-opt and AHSATS-D-CM based on the adaptive parameters. Time complexities of the two algorithms depend on the epoch length (Elen), the number of candidate solutions (CN), and the length of tabu list (TL). The epoch length (Elen) determines number of iterations, CN determines the number of neighborhood solution generations and the number of calculations of objective function



**Fig. 10.** Tour length changes resulting from increased iterations of AHSATS-2-opt and AHSATS-D-CM algorithms (gil262).

values, and  $TL$  determines the number of examinations of tabued solutions.

$$O(\text{AHSATS-2-opt}) = O(OLN \times Elen \times (CN + TL)) \quad (20)$$

where the  $OLN$  is the number of outside loops (number of decreases of temperature) of simulated annealing.

$$OLN = \frac{\ln t_{end} - \ln t_{start}}{\ln t_{cool}} \quad (21)$$

Since  $t_{start}$ ,  $t_{cool}$ , and  $t_{end}$  are constants,  $OLN$  is a constant.

$$\begin{aligned} O(\text{AHSATS-2-opt}) &= O(Elen \times (CN + TL)) \\ &= O(Elen \times CN + Elen \times TL) \\ &= O(N^{1.1} + N^{0.64}) \\ &= O(N^{1.1}) \end{aligned} \quad (22)$$

$Elen$ ,  $CN$ , and  $TL$  retain their previous definitions.

$$\begin{aligned} O(\text{AHSATS-D-CM}) &= O\left(Elen \times \sum_{k=1}^{OLN} \min(CN_{start} \times CN_{cool}^k, CN_{end}) + Elen \times TL\right) \\ &= O\left(CN_{start} \times Elen \times \sum_{k=1}^{OLN} \min\left(CN_{cool}^k, \frac{1538 + 1.35N^{1.14}}{1538 + N^{1.14}} + p_{end} - 1\right) + Elen \times TL\right) \\ &= O(N^{1.1} + N^{0.64}) \\ &= O(N^{1.1}) \end{aligned} \quad (23)$$

The complexities of the two algorithms are only  $O(N^{1.1})$  based on this set of adaptive parameters. The complexity of the algorithm is reasonable for a large scale TSP. Note that although the time complexities of the two algorithms are equal, AHSATS-D-CM has fewer iterations than AHSATS-2-opt.

## 4. Computational experiments

### 4.1. Numerical examples

Our algorithms, AHSATS-D-CM and AHSATS-2-opt, tested 64 benchmarks from TSPLIB, containing from 51 to 18,512 cities.

### 4.2. Running conditions

The benchmarks were tested with software implemented in C++ on a 3.10 GHz Windows 8 PC with 16GB RAM.

### 4.3. Setting of initial solutions

To investigate the influence of initial solutions on the quality of the solutions obtained by the two algorithms, we generated initial solutions using three different methods. The first is the nearest neighbor heuristic (NNH), in which the salesman starts from a city, then travels to the nearest city one by one until all cities are visited, and finally returns to the starting city. The second is random generation, which randomly generates the traversal order. The third, farthest heuristic (FH), is designed to maximize the tour length; the salesman starts from a city, then travels to the farthest city one by one until all cities are visited, and finally returns to the starting city. Each method generated two initial solutions, and each initial solution was optimized by AHSATS-2-opt and AHSATS-D-CM, for a total of six trials. In typical instances, the nearest neighbor heuristic obtains the shortest tour, random generation obtains a medium-length tour, and the farthest heuristic obtains the longest tour. If

the initial solution has an influence on the execution of the two algorithms, the experimental results will show significant differences in the quality of the three groups of final solutions which are derived from the three groups of initial solutions generated by the three methods.

### 4.4. Computational results

The experimental results are listed in Table 7. Columns “OPT”, “Best”, “Avg”, “Error<sub>best</sub>”, “Error<sub>avg</sub>” and “T” respectively denote the optimal tour length reported in TSPLIB, the tour length of the best solution among the six trials using our proposed hybrid algorithms, average tour length of the six trials, percentage gap between the shortest tour length and the optimal tour length reported in TSPLIB, percentage gap between the average tour length and the optimal tour length, and the average CPU time for all six trials. Both algorithms can obtain satisfactory solutions within a reasonable computational time. AHSATS-D-CM (AHSATS-2-opt) can always find the optimal solutions reported in TSPLIB

**Table 7**

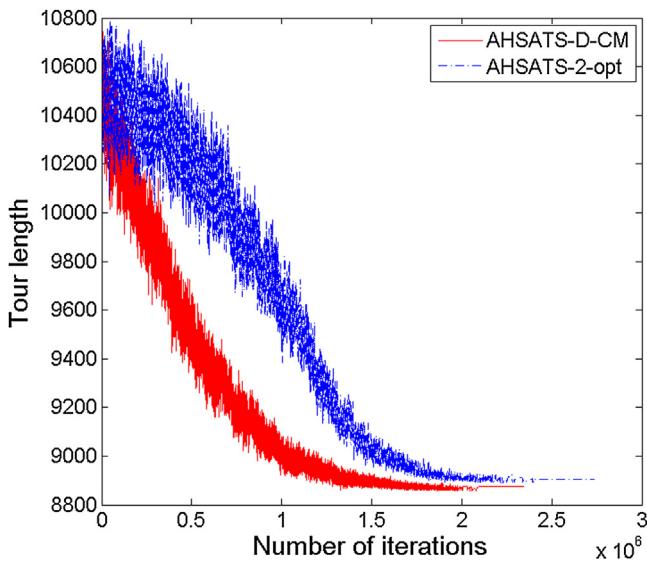
Running results of six trials for 64 benchmarks.

Instances	OPT	AHSATS-D-CM					AHSATS-2-opt				
		best	avg	Error <sub>best</sub>	Error <sub>avg</sub>	T(s)	best	avg	Error <sub>best</sub>	Error <sub>avg</sub>	T(s)
eil51	426	426	426.00	<b>0.000</b>	<b>0.000</b>	<b>2.729</b>	426	426.00	<b>0.000</b>	<b>0.000</b>	4.901
berlin52	7542	7542	7542.00	<b>0.000</b>	<b>0.000</b>	<b>4.645</b>	7542	7542.00	<b>0.000</b>	<b>0.000</b>	5.102
ncit60	6400	6400	6400.00	<b>0.000</b>	<b>0.000</b>	<b>2.850</b>	6400	6400.00	<b>0.000</b>	<b>0.000</b>	4.770
st70	675	675	675.00	<b>0.000</b>	<b>0.000</b>	<b>4.101</b>	675	675.00	<b>0.000</b>	<b>0.000</b>	6.592
eil76	538	538	538.00	<b>0.000</b>	<b>0.000</b>	<b>4.271</b>	538	538.00	<b>0.000</b>	<b>0.000</b>	7.696
pr76	108159	108159	108159.00	<b>0.000</b>	<b>0.000</b>	<b>5.597</b>	108159	108159.00	<b>0.000</b>	<b>0.000</b>	7.316
rat99	1211	1211	1211.00	<b>0.000</b>	<b>0.000</b>	<b>3.966</b>	1211	1211.00	<b>0.000</b>	<b>0.000</b>	9.900
rd100	7910	7910	7910.00	<b>0.000</b>	<b>0.000</b>	<b>4.636</b>	7910	7910.00	<b>0.000</b>	<b>0.000</b>	9.703
kroA100	21282	21282	21282.00	<b>0.000</b>	<b>0.000</b>	<b>4.740</b>	21282	21284.33	<b>0.000</b>	0.011	10.001
kroB100	22141	22141	22160.50	<b>0.000</b>	<b>0.088</b>	<b>5.078</b>	22141	22160.67	<b>0.000</b>	0.089	10.048
kroC100	20749	20749	20749.00	<b>0.000</b>	<b>0.000</b>	<b>4.403</b>	20749	20749.33	<b>0.000</b>	0.002	10.328
kroD100	21294	21294	21294.00	<b>0.000</b>	<b>0.000</b>	<b>4.901</b>	21294	21294.00	<b>0.000</b>	<b>0.000</b>	9.643
kroE100	22068	22068	22113.17	<b>0.000</b>	<b>0.205</b>	<b>5.270</b>	22076	22125.50	0.036	0.261	10.082
eil101	629	629	629.00	<b>0.000</b>	<b>0.000</b>	<b>5.669</b>	629	629.00	<b>0.000</b>	<b>0.000</b>	10.060
lin105	14379	14379	14379.00	<b>0.000</b>	<b>0.000</b>	<b>5.498</b>	14379	14382.67	<b>0.000</b>	0.026	11.114
pr107	44303	44303	44303.00	<b>0.000</b>	<b>0.000</b>	<b>4.526</b>	44303	44303.00	<b>0.000</b>	<b>0.000</b>	10.943
pr124	59030	59030	59037.67	<b>0.000</b>	0.013	<b>5.647</b>	59030	59030.00	<b>0.000</b>	<b>0.000</b>	12.616
bier127	118282	118282	118291.50	<b>0.000</b>	<b>0.008</b>	<b>15.154</b>	118282	118322.17	<b>0.000</b>	0.034	17.575
ch130	6110	6110	6113.17	<b>0.000</b>	<b>0.052</b>	<b>5.239</b>	6110	6119.83	<b>0.000</b>	0.161	12.563
pr136	96772	96785	96946.83	<b>0.013</b>	<b>0.181</b>	<b>5.521</b>	96920	96984.17	0.153	0.219	13.315
pr144	58537	58537	58581.17	<b>0.000</b>	0.075	<b>6.216</b>	58537	58563.50	<b>0.000</b>	<b>0.045</b>	13.836
ch150	6528	6528	6550.00	<b>0.000</b>	<b>0.337</b>	<b>6.273</b>	6549	6553.33	0.322	0.388	15.604
kroA150	26524	26524	26536.00	<b>0.000</b>	<b>0.045</b>	<b>6.260</b>	26525	26548.50	0.004	0.092	15.902
kroB150	26130	26132	26156.67	<b>0.008</b>	<b>0.102</b>	<b>6.687</b>	26135	26173.33	0.019	0.166	15.768
pr152	73682	73682	73754.67	<b>0.000</b>	<b>0.099</b>	<b>6.655</b>	73682	73825.00	<b>0.000</b>	0.194	16.021
u159	42080	42080	42238.33	<b>0.000</b>	0.376	<b>7.656</b>	42080	42186.83	<b>0.000</b>	<b>0.254</b>	15.681
rat195	2323	2328	2328.50	0.215	<b>0.237</b>	<b>7.979</b>	2323	2330.00	<b>0.000</b>	0.301	20.278
d198	15780	15791	15800.33	<b>0.070</b>	<b>0.129</b>	57.808	15798	15805.33	0.114	0.161	<b>56.943</b>
kroA200	29368	29383	29498.00	0.051	0.443	<b>8.520</b>	29368	29466.33	<b>0.000</b>	<b>0.335</b>	20.655
kroB200	29437	29437	29469.33	<b>0.000</b>	<b>0.110</b>	<b>8.148</b>	29445	29488.50	0.027	0.175	20.419
ts225	126643	126643	126704.67	<b>0.000</b>	<b>0.049</b>	<b>8.858</b>	126643	126773.33	<b>0.000</b>	0.103	22.492
pr226	80369	80369	80577.67	<b>0.000</b>	<b>0.260</b>	<b>24.144</b>	80467	80702.67	0.122	0.415	35.330
gil262	2378	2378	2380.67	<b>0.000</b>	<b>0.112</b>	<b>10.639</b>	2379	2384.17	0.042	0.259	28.041
pr264	49135	49135	49135.00	<b>0.000</b>	<b>0.000</b>	<b>14.823</b>	49135	49142.50	<b>0.000</b>	0.015	28.169
pr299	48191	48191	48251.83	<b>0.000</b>	<b>0.126</b>	<b>12.061</b>	48191	48255.83	<b>0.000</b>	0.135	31.685
lin318	42029	42175	42301.83	0.347	<b>0.649</b>	<b>12.529</b>	42148	42375.50	<b>0.283</b>	0.824	34.083
rd400	15281	15294	15339.33	<b>0.085</b>	<b>0.382</b>	<b>15.539</b>	15308	15363.33	0.177	0.539	43.153
fl417	11861	11867	11897.83	<b>0.051</b>	<b>0.311</b>	<b>70.618</b>	11872	11908.50	0.093	0.400	96.808
pr439	107217	107393	108039.17	<b>0.164</b>	<b>0.767</b>	<b>27.253</b>	107552	108808.83	0.312	1.485	47.664
pcb442	50778	50791	50979.33	<b>0.026</b>	<b>0.396</b>	<b>19.573</b>	50891	51060.00	0.223	0.555	48.789
u574	36905	36966	37056.67	<b>0.165</b>	<b>0.411</b>	<b>23.539</b>	37062	37215.83	0.425	0.842	64.493
rat575	6773	6791	6812.50	<b>0.266</b>	<b>0.583</b>	<b>24.201</b>	6807	6822.17	0.502	0.726	60.184
u724	41910	42131	42173.17	<b>0.527</b>	<b>0.628</b>	<b>32.272</b>	42199	42261.33	0.690	0.838	82.079
rat783	8806	8838	8865.33	<b>0.363</b>	<b>0.674</b>	<b>35.613</b>	8878	8900.00	0.818	1.067	86.248
pr1002	259045	260540	261631.17	<b>0.577</b>	<b>0.998</b>	<b>47.886</b>	261538	262182.17	0.962	1.211	120.524
pcb1173	56892	57092	57261.33	<b>0.352</b>	<b>0.649</b>	<b>58.969</b>	57400	57550.50	0.893	1.157	129.962
d1291	50801	51187	51349.50	<b>0.760</b>	<b>1.080</b>	197.472	51238	51672.33	0.860	1.715	<b>179.793</b>
r1304	252948	254967	255948.50	<b>0.798</b>	<b>1.186</b>	<b>64.243</b>	255582	257798.00	1.041	1.917	151.194
r1323	270199	271092	272013.33	<b>0.330</b>	<b>0.671</b>	<b>68.257</b>	272881	273648.00	0.993	1.276	153.785
fl1400	20127	20227	20481.33	<b>0.497</b>	<b>1.760</b>	<b>527.347</b>	20354	20570.00	1.128	2.201	616.704
fl1577	22249	22279	22513.17	0.135	<b>1.187</b>	<b>81.542</b>	22270	22579.67	<b>0.094</b>	1.486	187.295
d1655	62128	62823	62950.33	<b>1.119</b>	<b>1.324</b>	235.178	62968	63101.00	1.352	1.566	<b>211.284</b>
vm1748	336556	339669	340478.50	<b>0.925</b>	<b>1.165</b>	<b>99.985</b>	339695	341282.67	0.933	1.404	204.661
rl1889	316536	319798	322133.83	<b>1.031</b>	<b>1.768</b>	<b>101.979</b>	321533	322752.17	1.579	1.964	233.492
d2103	80450	80603	81392.33	<b>0.190</b>	<b>1.171</b>	299.784	80920	81705.33	0.584	1.560	<b>269.169</b>
u2319	234256	235274	235391.33	<b>0.435</b>	<b>0.485</b>	<b>132.388</b>	235749	235894.17	0.637	0.699	297.043
pr2392	378032	382922	383621.67	<b>1.294</b>	<b>1.479</b>	<b>133.704</b>	383429	384559.17	1.428	1.727	285.931
pcb3038	137694	138870	139185.67	<b>0.854</b>	<b>1.083</b>	<b>168.122</b>	139116	139329.33	1.033	1.188	366.544
fnl4461	182566	184394	184625.00	<b>1.001</b>	<b>1.128</b>	<b>275.655</b>	185488	185626.50	1.601	1.676	551.400
r15934	556045	564590	566692.33	<b>1.537</b>	<b>1.915</b>	<b>379.243</b>	565346	567442.83	1.673	2.050	823.702
pla7397	23260728	23745871	23935651.17	<b>2.086</b>	<b>2.902</b>	<b>661.815</b>	23970962	24028928.17	3.088	3.303	1109.000
usa13509	19982859	20304083	20359728.67	<b>1.607</b>	<b>1.886</b>	<b>1119.905</b>	20389514	20432729.67	2.035	2.251	2188.455
brd14051	469385	475279	478872.33	<b>1.256</b>	<b>2.021</b>	<b>1230.269</b>	480059	481100.50	2.274	2.496	2367.851
d18512	645238	656621	659364.83	<b>1.764</b>	<b>2.189</b>	3328.670	660289	664009.33	2.333	2.909	<b>2969.112</b>
Average				0.327	0.561				0.483	0.732	

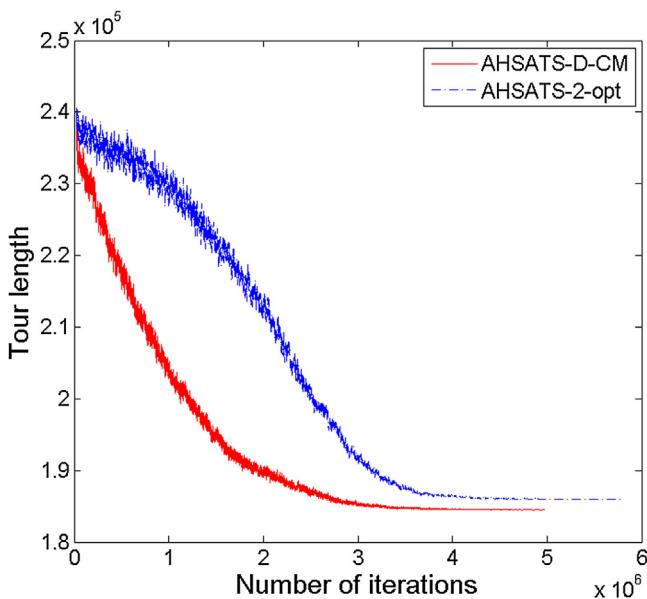
Bold numbers are the more excellent values obtained by the two algorithms.

for 15 (12) out of the 64 benchmarks, and optimal solutions for 30 (26) benchmarks can be found in six trials. The average Error<sub>best</sub> and average Error<sub>avg</sub> are 0.327 (0.483) and 0.561 (0.732) for AHSATS-D-CM (AHSATS-2-opt), respectively. From the comparison between AHSATS-D-CM and AHSATS-2-opt, we can

see that AHSATS-D-CM is more accurate than AHSATS-2-opt. Although AHSATS-D-CM does not have a significant advantage over AHSATS-D-CM when the scale is relatively small, the superiority of AHSATS-D-CM over AHSATS-2-opt becomes more significant when the scale increases. Moreover, for most of the benchmarks,



**Fig. 11.** Tour length changes resulting from increased iterations of AHSATS-2-opt and AHSATS-D-CM algorithms (rat783).



**Fig. 12.** Tour length changes resulting from increased iterations of AHSATS-2-opt and AHSATS-D-CM algorithms (fnl4461).

AHSATS-D-CM requires much less computation time compared to AHSATS-2-opt.

Figs. 10–12 show the changes in tour length for instances gil262, rat783, and fnl4461 that occur when the number of iterations increases for both algorithms. It is evident that both AHSATS-2-opt and AHSATS-D-CM exhibit a fast decrease rate and a clear convergence process. However, the number of iterations executed by AHSATS-D-CM is fewer than that of AHSATS-2-opt. AHSATS-D-CM's convergence rate is faster than that of AHSATS-2-opt.

Finally, in Table 8, the columns “NNH,” “random,” and “FH” respectively denote the three methods of generating initial solutions (nearest neighbor heuristic, random generation, and farthest heuristic). The rows “average” and “NM” denote the average gap and number of minimum gaps achieved based on the three

groups of initial solutions. This table shows the relationship between the initial solutions and the results of the trials. For AHSATS-D-CM (AHSATS-2-opt), the average gaps are 0.519, 0.580, and 0.583 (0.765, 0.714, and 0.718) and the number of minimum gaps achieved are 35, 32, and 31 (30, 32, and 32), respectively, for the three groups of initial solutions. There is no significant difference in the results based on the three groups of initial solutions. Thus, we cannot conclude that the initial solutions' quality has an influence on the final solutions obtained by the two algorithms.

From the analysis above, we can draw the following conclusions:

1. The hybrid simulated – tabu search algorithms have a fast decrease rate and a clear convergence process.
2. The quality of the initial solutions has almost no influence on the results of the hybrid algorithms.
3. The dynamic neighborhood structure is more accurate and efficient than the classical 2-opt because it can simultaneously reduce the number of candidate solutions that are generated, reduce the total number of iterations by improving the probability of acceptance, and ensure the quality of the solutions during the iterative process.
4. The adaptive parameters are appropriate for almost all numerical examples.

#### 4.5. Comparisons of the proposed algorithm with improved traditional meta-heuristic algorithms in recent works

In this subsection, we compare our algorithm AHSATS-D-CM with seven recent improved algorithms based on traditional meta-heuristic strategies—SA, TS, GA, ACO, and PSO. These include the Improved Ant Colony Optimization Algorithm (IACO) [36], Genetic Algorithm with Greedy Sub Tour Mutation (GA-GSTM) [29], Genetic Simulated Annealing Ant Colony System with Particle Swarm Optimization (GSAACPSO) [49], Adaptive Simulated Annealing Algorithm with Greedy Search (ASA-GS) [21], Hybrid Genetic Algorithm (HGA) [31], Ant Colony Extended (ACE) [37], and hybrid algorithm of Particle Swarm Optimization, Ant Colony Optimization and 3-Opt (PSO-ACO-3Opt) [75]. The results are listed in Table 9.

From the table, we can see that the quality of solutions obtained by our algorithm AHSATS-D-CM exceeds those reported in other studies for most tested benchmarks in terms of the best gap ( $\text{error}_{\text{best}}$ ) and average gap ( $\text{error}_{\text{avg}}$ ). AHSATS-D-C obtains no worse solutions than those obtained by IACO, GA-GSTM, GSAACPSO, and PSO-ACO-3Opt for all benchmarks tested. ASA-GS, HGA, and ACE found better solutions for only a few benchmarks. The proposed AHSATS-D-CM is obviously more accurate than the algorithms selected for comparison.

#### 4.6. Comparisons of the proposed algorithm with other algorithms

Table 10 lists the rankings of 59 algorithms from the DIMACS implementation challenge (<http://www.research.att.com/~dsj/chsp/>). These include tour construction algorithms, simple local search algorithms, Lin-Kernighan implementations and variants, repeated local search algorithms, algorithms that use Chained Lin-Kernighan as a subroutine, and metaheuristics for solving 10 benchmarks (pr1002, pcb1173, d1291, rl1304, rl1323, fl1400, fl1577, rl1889, d2103, and pr2392). For a detailed explanation of the table contents, please refer to Ref. [65].

AHSATS-D-CM and AHSATS-2-opt are ranked in the 9th and 13th places, respectively, among the 59 algorithms. Both of the algorithms are ranked higher than all tour construction heuristics and all simple local search algorithms. Only a few Lin-Kernighan-based Variable Neighborhood Search implementations and one

**Table 8**

Experimental results of different initial solutions generated by three methods.

Instances	AHSATS-D-CM						AHSATS-2-opt					
	NNH		random		FH		NNH		random		FH	
	average	error <sub>avg</sub>	average	error <sub>avg</sub>	average	error <sub>avg</sub>	average	error <sub>avg</sub>	average	error <sub>avg</sub>	average	error <sub>avg</sub>
eil51	426.0	<b>0.000</b>	426.0	<b>0.000</b>	426.0	<b>0.000</b>	426.0	<b>0.000</b>	426.0	<b>0.000</b>	426.0	<b>0.000</b>
berlin52	7542.0	<b>0.000</b>	7542.0	<b>0.000</b>	7542.0	<b>0.000</b>	7542.0	<b>0.000</b>	7542.0	<b>0.000</b>	7542.0	<b>0.000</b>
ncit60	6400.0	<b>0.000</b>	6400.0	<b>0.000</b>	6400.0	<b>0.000</b>	6400.0	<b>0.000</b>	6400.0	<b>0.000</b>	6400.0	<b>0.000</b>
st70	675.0	<b>0.000</b>	675.0	<b>0.000</b>	675.0	<b>0.000</b>	675.0	<b>0.000</b>	675.0	<b>0.000</b>	675.0	<b>0.000</b>
eil76	538.0	<b>0.000</b>	538.0	<b>0.000</b>	538.0	<b>0.000</b>	538.0	<b>0.000</b>	538.0	<b>0.000</b>	538.0	<b>0.000</b>
pr76	108159.0	<b>0.000</b>	108159.0	<b>0.000</b>	108159.0	<b>0.000</b>	108159.0	<b>0.000</b>	108159.0	<b>0.000</b>	108159.0	<b>0.000</b>
rat99	1211.0	<b>0.000</b>	1211.0	<b>0.000</b>	1211.0	<b>0.000</b>	1211.0	<b>0.000</b>	1211.0	<b>0.000</b>	1211.0	<b>0.000</b>
rd100	7910.0	<b>0.000</b>	7910.0	<b>0.000</b>	7910.0	<b>0.000</b>	7910.0	<b>0.000</b>	7910.0	<b>0.000</b>	7910.0	<b>0.000</b>
kroA100	21282.0	<b>0.000</b>	21282.0	<b>0.000</b>	21282.0	<b>0.000</b>	21282.0	<b>0.000</b>	21282.0	<b>0.000</b>	21289.0	0.033
kroB100	22141.0	<b>0.000</b>	22141.0	<b>0.000</b>	22199.5	0.264	22200.0	0.266	22141.0	<b>0.000</b>	22141.0	<b>0.000</b>
kroC100	20749.0	<b>0.000</b>	20749.0	<b>0.000</b>	20749.0	<b>0.000</b>	20749.0	<b>0.000</b>	20750.0	0.005	20749.0	<b>0.000</b>
kroD100	21294.0	<b>0.000</b>	21294.0	<b>0.000</b>	21294.0	<b>0.000</b>	21294.0	<b>0.000</b>	21294.0	0.000	21294.0	<b>0.000</b>
kroE100	22114.5	0.211	22094.5	<b>0.120</b>	22130.5	0.283	22147.5	0.360	22110.5	<b>0.193</b>	22118.5	0.229
eil101	629.0	<b>0.000</b>	629.0	<b>0.000</b>	629.0	<b>0.000</b>	629.0	<b>0.000</b>	629.0	<b>0.000</b>	629.0	<b>0.000</b>
lin105	14379.0	<b>0.000</b>	14379.0	<b>0.000</b>	14379.0	<b>0.000</b>	14379.0	<b>0.000</b>	14379.0	<b>0.000</b>	14390.0	0.077
pr107	44303.0	<b>0.000</b>	44303.0	<b>0.000</b>	44303.0	<b>0.000</b>	44303.0	<b>0.000</b>	44303.0	<b>0.000</b>	44303.0	<b>0.000</b>
pr124	59030.0	<b>0.000</b>	59053.0	0.039	59030.0	<b>0.000</b>	59030.0	<b>0.000</b>	59030.0	<b>0.000</b>	59030.0	<b>0.000</b>
bier127	118295.0	0.011	118297.5	0.013	118282.0	<b>0.000</b>	118297.5	<b>0.013</b>	118316.5	0.029	118352.5	0.060
ch130	6115.0	0.082	6112.0	<b>0.033</b>	6112.5	0.041	6125.5	0.254	6121.5	0.188	6112.5	<b>0.041</b>
pr136	96919.5	0.152	97020.0	0.256	96901.0	<b>0.133</b>	96920.0	<b>0.153</b>	97067.5	0.305	96965.0	0.199
pr144	58563.5	<b>0.045</b>	58590.0	0.091	58590.0	0.091	58563.5	<b>0.045</b>	58563.5	<b>0.045</b>	58563.5	<b>0.045</b>
ch150	6552.5	0.375	6553.5	0.391	6544.0	<b>0.245</b>	6552.5	0.375	6556.0	0.429	6551.5	<b>0.360</b>
kroA150	26530.5	0.025	26524.5	<b>0.002</b>	26553.0	0.109	26568.0	0.166	26525.0	<b>0.004</b>	26552.5	0.107
kroB150	26144.5	<b>0.055</b>	26174.5	0.170	26151.0	0.080	26177.5	0.182	26183.5	0.205	26159.0	<b>0.111</b>
pr152	73750.0	0.092	73682.0	<b>0.000</b>	73832.0	0.204	73767.5	<b>0.116</b>	73827.5	0.197	73880.0	0.269
u159	42239.0	0.378	42238.0	<b>0.375</b>	42238.0	<b>0.375</b>	42238.0	0.375	42242.5	0.386	42080.0	<b>0.000</b>
rat195	2328.0	<b>0.215</b>	2328.0	<b>0.215</b>	2329.5	0.280	2332.5	0.409	2331.5	0.366	2326.0	<b>0.129</b>
d198	15801.0	0.133	15798.5	<b>0.117</b>	15801.5	0.136	15811.5	0.200	15802.0	<b>0.139</b>	15802.5	0.143
kroA200	29456.5	0.301	29616.0	0.844	29421.5	<b>0.182</b>	29429.0	<b>0.208</b>	29525.5	0.536	29444.5	0.260
kroB200	29482.0	0.153	29461.5	<b>0.083</b>	29464.5	0.093	29469.0	<b>0.109</b>	29517.0	0.272	29479.5	0.144
ts225	126713.0	0.055	126700.5	<b>0.045</b>	126700.5	<b>0.045</b>	126719.5	0.060	126713.0	<b>0.055</b>	126887.5	0.193
pr226	80496.5	0.159	80816.5	0.557	80420.0	<b>0.063</b>	80753.0	0.478	80673.5	<b>0.379</b>	80681.5	0.389
gil262	2378.5	<b>0.021</b>	2380.5	0.105	2383.0	0.210	2379.0	<b>0.042</b>	2383.0	0.210	2390.5	0.526
pr264	49135.0	<b>0.000</b>	49135.0	<b>0.000</b>	49135.0	0.000	49135.0	<b>0.000</b>	49135.0	<b>0.000</b>	49157.5	0.046
pr299	48248.0	0.118	48275.5	0.175	48232.0	<b>0.085</b>	48250.0	0.122	48309.0	0.245	48208.5	<b>0.036</b>
lin318	42267.5	<b>0.567</b>	42282.5	0.603	42355.5	0.777	42376.0	0.826	42407.0	0.899	42343.5	<b>0.748</b>
rd400	15297.0	<b>0.105</b>	15363.5	0.540	15357.5	0.501	15340.5	<b>0.389</b>	15357.0	0.497	15392.5	0.730
fl417	11901.5	0.341	11922.5	0.519	11869.5	<b>0.072</b>	11904.0	0.363	11923.5	0.527	11898.0	<b>0.312</b>
pr439	107992.0	0.723	108138.0	0.859	107987.5	<b>0.719</b>	109208.5	1.857	107606.5	<b>0.363</b>	109611.5	2.233
pcb442	50949.5	<b>0.338</b>	51028.0	0.492	50960.5	0.359	51109.0	0.652	51044.0	0.524	51027.0	<b>0.490</b>
u574	37017.5	<b>0.305</b>	37072.5	0.454	37080.0	0.474	37257.5	0.955	37150.5	<b>0.665</b>	37239.5	0.906
rat575	6797.0	<b>0.354</b>	6819.0	0.679	6821.5	0.716	6820.0	<b>0.694</b>	6823.5	0.746	6823.0	0.738
u724	42164.0	0.606	42160.0	<b>0.597</b>	42195.5	0.681	42286.5	0.898	42278.0	0.878	42219.5	<b>0.738</b>
rat783	8863.0	0.647	8875.0	0.784	8858.0	<b>0.591</b>	8887.5	<b>0.926</b>	8906.0	1.136	8906.5	1.141
pr1002	261325.0	<b>0.880</b>	262018.0	1.148	261550.5	0.967	262134.0	1.192	262185.0	<b>1.212</b>	262227.5	1.229
pcb1173	57351.5	0.808	57234.0	0.601	57198.5	<b>0.539</b>	57536.0	1.132	57650.0	1.332	57465.5	<b>1.008</b>
d1291	51380.5	1.141	51456.0	1.289	51212.0	<b>0.809</b>	51480.0	<b>1.337</b>	51807.0	1.980	51730.0	1.829
r11304	255280.0	<b>0.922</b>	256353.0	1.346	256212.5	1.291	259480.5	2.583	256737.5	<b>1.498</b>	257176.0	1.671
r11323	271651.0	<b>0.537</b>	272202.0	0.741	272187.0	0.736	274361.5	1.541	273196.5	<b>1.109</b>	273386.0	1.180
fl1400	20443.5	1.573	20436.5	<b>1.538</b>	20564.0	2.171	20722.5	2.959	20544.5	2.074	20443.0	<b>1.570</b>
fl1577	22468.0	0.984	22465.5	<b>0.973</b>	22606.0	1.605	22701.0	2.032	22450.5	<b>0.906</b>	22587.5	1.521
d1655	63059.0	1.499	62843.0	<b>1.151</b>	62949.0	1.321	63162.0	1.664	62985.0	<b>1.379</b>	63156.0	1.655
vm1748	341426.5	1.447	340045.0	1.037	339964.0	<b>1.013</b>	340150.5	<b>1.068</b>	342950.0	1.900	340747.5	1.245
r11889	320867.0	<b>1.368</b>	321351.0	1.521	324183.5	2.416	323007.0	2.044	322098.0	<b>1.757</b>	323151.5	2.090
d2103	81105.5	<b>0.815</b>	81162.5	0.886	81909.0	1.814	81631.5	1.469	81973.5	1.894	81511.0	<b>1.319</b>
u2319	235470.5	0.518	235306.5	<b>0.448</b>	235397.0	0.487	235962.5	0.728	235807.0	<b>0.662</b>	235913.0	0.707
pr2392	384038.5	1.589	383133.5	<b>1.349</b>	383693.0	1.497	384458.0	<b>1.700</b>	384671.5	1.756	384548.0	1.724
pcb3038	139182.0	1.081	139323.0	1.183	139052.0	<b>0.986</b>	139310.0	1.174	139334.0	<b>1.191</b>	139344.0	1.198
fnl4461	184836.0	1.243	184494.5	<b>1.056</b>	184544.5	1.084	185660.0	1.695	185720.5	1.728	185499.0	<b>1.607</b>
rl5934	566100.5	<b>1.808</b>	567279.5	2.020	566697.0	1.916	569663.5	2.449	566453.5	1.872	566211.5	<b>1.828</b>
pla7397	23794414.0	<b>2.294</b>	24068214.5	3.471	23944325.0	2.939	24046177.5	3.377	23993299.5	<b>3.149</b>	24047307.5	3.382
usa13509	20334728.0	<b>1.761</b>	20377160.0	1.973	20367298.0	1.924	20419600.5	2.186	20467951.5	2.428	20410637.0	<b>2.141</b>
brd14051	479844.5	2.228	478973.0	2.043	477799.5	<b>1.793</b>	481400.5	2.560	481321.0	2.543	480580.0	<b>2.385</b>
d18512	659263.5	<b>2.174</b>	659464.5	2.205	659366.5	2.190	661781.5	<b>2.564</b>	664105.0	2.924	666141.5	3.240
Average	0.519		0.580		0.583		0.765		0.714		0.718	
NM	35		32		31		30		32		32	

Bold numbers are the minimum average errors using the three methods "NNH", "random" and "FH" for each of the two hybrid algorithms, AHSATS-D-CM and AHSATS-2-opt.

metaheuristic algorithm (HBMO) are ranked higher than our algorithms, because our algorithms

**Table 9**

Comparison with meta-heuristics reported in literature.

Instances	Error <sub>best</sub> (%)							Error <sub>avg</sub> (%)								
	IACO	GA-GSTM	GSAACPSO	ASA-GS	HGA	ACE	PSO-ACO-3Opt	AHSATS-D-CM	IACO	GA-GSTM	GSAACPSO	ASA-GS	HGA	ACE	PSO-ACO-3Opt	AHSATS-D-CM
eil51	<b>0.000</b>		0.235	0.674	0.674	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	0.763		0.298	0.674	0.749	0.192	0.106	<b>0.000</b>
berlin52	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	0.031	0.031	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	0.438	<b>0.000</b>	<b>0.000</b>	0.031	0.031	0.014	0.016	<b>0.000</b>
ncit64			<b>0.000</b>					<b>0.000</b>			<b>0.000</b>					<b>0.000</b>
st70	<b>0.000</b>			0.313	0.313	<b>0.000</b>	0.148	<b>0.000</b>	1.311			0.313	0.354	0.210	0.474	<b>0.000</b>
eil76	<b>0.000</b>		<b>0.000</b>	1.184	1.184	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	1.199		0.409	1.184	1.498	0.058	0.056	<b>0.000</b>
pr76				<b>0.000</b>	<b>0.000</b>	<b>0.000</b>					<b>0.000</b>	0.090	0.085			<b>0.000</b>
rat99				0.680	0.680	<b>0.000</b>	<b>0.000</b>					0.701	0.904	0.189	0.278	<b>0.000</b>
rd100			<b>0.000</b>	0.005	0.005							0.981	0.005	0.044		<b>0.000</b>
kroA100	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	0.016	0.016	<b>0.000</b>	0.089	<b>0.000</b>	0.664	1.184	0.416	0.016	0.143	0.078	0.766	<b>0.000</b>
kroB100		<b>0.000</b>		–				<b>0.000</b>			0.641	–				<b>0.088</b>
kroC100		<b>0.000</b>		0.009	0.008			<b>0.000</b>			0.626	0.009	0.305			<b>0.000</b>
kroD100		0.070	0.001	0.001				<b>0.000</b>			1.533	0.033	0.238			<b>0.000</b>
kroE100		<b>0.000</b>	0.174					<b>0.000</b>			0.523	<b>0.201</b>				0.205
eil101		0.159	1.783	1.782	<b>0.000</b>	0.318	<b>0.000</b>				0.990	1.831	2.515	0.734	0.588	<b>0.000</b>
lin105	<b>0.000</b>	<b>0.000</b>	0.028	0.028	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	0.651		0.190	0.028	0.305	0.045	0.001	<b>0.000</b>	
pr107		–	<b>0.000</b>					<b>0.000</b>			–	0.087				<b>0.000</b>
pr124			0.001	0.001				<b>0.000</b>				<b>0.001</b>	0.109			0.013
bier127		<b>0.000</b>	0.010					<b>0.000</b>			0.964	0.057				<b>0.008</b>
ch130		0.507	0.012	0.012	<b>0.000</b>			<b>0.000</b>			1.565	0.182	0.332	0.719		<b>0.052</b>
pr136		0.201	0.014					<b>0.013</b>			0.317	0.256				<b>0.181</b>
pr144	<b>0.000</b>	<b>0.000</b>	–	<b>0.000</b>				<b>0.000</b>	0.246	1.081	0.015	<b>0.000</b>				0.075
ch150	0.460	<b>0.000</b>	0.044	0.044	<b>0.000</b>	0.153	<b>0.000</b>	<b>0.000</b>		0.636	0.547	<b>0.181</b>	0.455	0.337	0.551	0.337
kroA150		<b>0.000</b>	0.003	0.003			<b>0.000</b>			1.415	0.055	0.278				<b>0.045</b>
kroB150	0.964	<b>0.000</b>	0.041	<b>0.000</b>				0.008		1.762	1.218	0.184	<b>0.788</b>			0.102
pr152	0.770		0.002	0.002	<b>0.000</b>		<b>0.000</b>			1.529	<b>0.017</b>	0.114	0.115			0.099
u159		0.744		<b>0.000</b>			<b>0.000</b>			1.620		0.758			<b>0.285</b>	0.376
rat195	0.603		0.957	1.043			<b>0.215</b>			1.843		1.078	1.421			<b>0.237</b>
d198	0.387		0.321	0.741	<b>0.000</b>			0.070		1.219		0.414	1.161	0.211		<b>0.129</b>
kroA200	0.868	0.051	0.148	<b>0.005</b>		0.341	0.051			1.543	1.262	<b>0.240</b>	0.309		0.947	0.443
kroB200		0.353	0.228	0.046			<b>0.000</b>			2.032		0.259	0.497			<b>0.110</b>
ts225	0.253		0.002	1.184			<b>0.000</b>			0.499	<b>0.002</b>	1.305				0.049
pr226	0.724		0.215	0.083			<b>0.000</b>			1.529		0.396	<b>0.206</b>			0.260
gil262		0.658		<b>0.000</b>			<b>0.000</b>			0.867				0.507		<b>0.112</b>
pr264		<b>0.000</b>	0.033				<b>0.000</b>					0.008	0.058			<b>0.000</b>
pr299	1.233		0.162	2.638			<b>0.000</b>		1.233			0.281	3.251			<b>0.126</b>
lin318	0.983	1.090	0.661	1.416			<b>0.347</b>		3.310	2.317		0.844	2.018			<b>0.649</b>
rd400			0.456	5.030			<b>0.085</b>					0.974	5.647			<b>0.382</b>
f1417		0.669					<b>0.051</b>					1.541				<b>0.311</b>
pr439		2.614	2.755				<b>0.164</b>					2.806	3.724			<b>0.767</b>
pcb442	2.050		0.563	4.128			<b>0.026</b>		2.776			0.967	4.408			<b>0.396</b>
u574		0.887					<b>0.165</b>					1.259				<b>0.411</b>
rat575		1.742	1.463				<b>0.266</b>			2.375		1.946				<b>0.583</b>
u724		0.870					<b>0.527</b>					1.337				<b>0.628</b>
rat783		2.067	1.685		0.704		<b>0.363</b>			3.103		2.001	1.484			<b>0.674</b>
pr1002			1.724				<b>0.577</b>					2.019				<b>0.998</b>
pcb1173			1.527				<b>0.352</b>					1.632				<b>0.649</b>
d1291			1.870				<b>0.760</b>					2.857				<b>1.080</b>
r11323		2.755	0.653				<b>0.330</b>			3.694		1.201				<b>0.671</b>
fl1400		2.315	2.586		0.999		<b>0.497</b>			6.075		3.255	1.811			<b>1.760</b>
d1655		3.256	2.427				<b>1.119</b>			5.622		3.264				<b>1.324</b>
vm1748			1.747				<b>0.925</b>					2.185				<b>1.165</b>
u2319			0.896				<b>0.435</b>					1.062				<b>0.485</b>
pcb3038			2.214				<b>0.854</b>					2.577				<b>1.083</b>
fnl4461			2.405				<b>1.001</b>					2.653				<b>1.128</b>
rl5934			2.938				<b>1.537</b>					3.487				<b>1.915</b>
pla7397			3.616				<b>2.086</b>					3.894				<b>2.902</b>
usa13509			4.087				<b>1.607</b>					4.145				<b>1.886</b>
brd14051			3.459				<b>1.256</b>					3.702				<b>2.021</b>
d18512			3.544				<b>1.764</b>					3.752				<b>2.189</b>

Bold numbers represent the optimal values among those obtained by all different algorithms in terms of average error and best error.

benchmarks. Note that a fair comparison in terms of computational efficiency is difficult, because computational speed is affected by the compiler and the hardware that are used. Therefore, the comparisons against the algorithms from the literature were performed only terms of solution quality

## 5. Conclusion

This paper develops an adaptive hybrid meta-heuristic algorithm that combines simulated annealing and tabu search algorithms with a dynamic neighborhood structure to solve

the Traveling Salesman Problem (TSP). The experimental results demonstrated that the proposed algorithm can obtain satisfactory solutions within a reasonable time. Moreover, the proposed algorithm can overcome the individual disadvantages of simulated annealing and tabu search. The new hybrid algorithm provides a fast decrease rate and a clear convergence process, and is independent on initial solutions. Furthermore, compared with the classical 2-opt neighborhood, the dynamic neighborhood can significantly reduce computational time by decreasing the number of calculations and simultaneously improve solution quality. Finally, the

**Table 10**

Ranking of the algorithms based on the average quality (%).

Ranks.	Algorithms	Error(%)	Ranks	Algorithms	Error (%)
1	HBMO [43]	0	31	3opt-B [68]	3.635
2	Tour Merging [15]	0.004	32	Concorde-Lin-Kernighan [15]	3.916
3	ILK-NYYY [71]	0.009	33	4-Hyperopt [12]	4.062
4	Iterated Lin – Kernighan by Johnson [12]	0.041	34	Applegate Lin Kernighan [15]	4.308
5	Keld Helsgaun's Multi-Trial Variant on Lin-Kernighan [14]	0.064	35	3-Hyperopt [12]	4.559
6	Iterated-3-Opt by Johnson [12]	0.413	36	GENIUS [70]	4.685
7	Augmented-LK [71]	0.42	37	Tabu-search-2opt-2opt [22]	4.83
8	Applegate-Cook-Rohe Chained Lin-Kernighan [15]	0.567	38	Memetic SOM [74]	5.775
9	<b>AHSATS-D-CM</b>	0.596	39	2.5opt-B [68]	5.93
10	Iterated Lin-Kernighan by Neto [15]	0.642	40	2opt-J [12]	6.113
11	Multi-Level Lin-Kernighan Implementations [73]	0.743	41	2-Hyperopt [12]	6.218
12	Concorde Chain Lin-Kernighan [15]	0.88	42	Held Karp Christofides [71]	6.496
13	<b>AHSATS-2-opt</b>	0.956	43	2opt-B [68]	6.83
14	Helsgaun Lin-Kernighan [14]	0.969	44	GENI [70]	8.05
15	Tabu-search-SC-DB [22]	1.034	45	CCA [71]	10.43
16	Expanding Neighborhood Search GRASP [65]	1.181	46	Clarke-Wright[69]	11.26
17	Lin-Kernighan-with-HK-Christo-starts [71]	1.202	47	2opt-C [15]	14.56
18	Tabu-search-LK-DB [22]	1.241	48	FI [68]	16.64
19	Variable-Neighborhood-Search-Using-3-Hyperopt [71]	1.292	49	RI [5]	17.47
20	Balas-Simonetti-Dynamic-Programming-Heuristic [71]	1.308	50	Boruvka [15]	17.76
21	Johnson Lin-Kernighan [12]	1.41	51	CHCI [68]	17.96
22	ASA-GS [21]	1.672	52	C-Greedy [15]	18.23
23	Stem-Cycle Ejection Chain [71]	1.702	53	B-Greedy [71]	18.708
24	LK-NYYY [71]	1.706	54	Q-Boruvka [15]	19.6
25	Neto Lin-Kernighan [72]	1.804	55	NN [15]	24.85
26	Variable-Neighborhood-Search-Using-2-Hyperopt [71]	1.993	56	Spacefilling [71]	47.545
27	Tabu-search-Stem – Cycle –SC [22]	2.034	57	Best-Way Strip [71]	49.405
28	Tabu-search-LK-LK [22]	2.2	58	FRP [68]	74.36
29	Tabu-search-2opt-Double Bridge [22]	3.11	59	Strip [71]	75.565
30	3opt-J [12]	3.5			

adaptive parameters are appropriate for solving almost all benchmark test examples.

## 6. Future research

There are several areas for future research. First, we anticipate that the dynamic neighborhood structure will be adaptable to other evolutionary algorithms as a local search strategy or a mutation operator. These evolutionary algorithms may include genetic algorithms, ant colony optimization, particle swarm optimization, and honey bee mating optimization. Furthermore, the idea of the dynamic neighborhood is available for other neighborhoods, such as 3-opt, 4-opt, and even variable neighborhoods. In other future works, we plan to apply the dynamic neighborhood concept to other types of local search problems. Finally, our algorithms can be processed in parallel; therefore we will develop parallel algorithms based on the proposed algorithms in order to solve numerical examples with larger scales.

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