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Mechanism design for first-mile ridesharing based on personalized requirements part II: Solution algorithm for large-scale problems

Zheyong Bian, Xiang Liu*

Department of Civil and Environmental Engineering, Rutgers, The State University of New Jersey, 500 Bartholomew Road, Room 428D, Piscataway, NJ 08854-8018, United States

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ABSTRACT

Ridesharing provides travelers with a low-cost and convenient first-mile mobility service. Our Part I paper designed a mechanism to incentivize more travelers to participate in the first-mile ridesharing service accounting for passengers' personalized requirements on inconvenience attributes of ridesharing. In order to address the computational challenge of obtaining the mechanism for large-scale transportation networks, this paper develops a novel heuristic algorithm, called the Solution Pooling Approach (SPA) for efficiently solving large-scale mechanism design problems in the first-mile ridesharing context. This paper also extends the SPA to solve generalized mechanism design problems, analyzes specific circumstances under which the SPA can sustain the game-theoretic properties, including "individual rationality" and "incentive compatibility", and identifies its limitations. For the particular application in first-mile ridesharing, the SPA maintains the properties of "individual rationality" and "incentive compatibility". Numerical experimental results show that the SPA can address the complex first-mile ridesharing service mechanism design problem in a computationally viable and efficient manner.

1. Introduction

Americans take 11 billion trips annually on public transportation, a 40% increase since 1995 (American Public Transportation Association, 2016). The American public transportation industry faces an ongoing challenge – transit hub accessibility. This challenge is also known as the "*first mile*" bottleneck. Several studies have found that travelers' choice of public transportation is significantly hindered by the accessibility to transit hubs (Krygsman et al., 2004; Rietveld, 2000). Ridesharing provides travelers with a low-cost and convenient mobility service, and can reduce congestion on roads, emissions, and parking demands (Furuhata et al., 2013; Cici et al., 2014; Kuhr et al., 2017). Thus, ridesharing is a potential solution to address first-mile transit accessibility (Lesh, 2013; Shaheen and Chan, 2016; Alemi and Rodier, 2016; Masoud et al., 2017a; Bian and Liu, 2017).

In order to incentivize more travelers to participate in the first-mile ridesharing service, our Part I paper proposed an incentive mechanism based on passengers' personalized requirements on inconvenience attributes, including the number of shared co-riders, extra in-vehicle travel time due to detours, and extra waiting time at the transit hub due to early arrival.

* Corresponding author.
E-mail address: xiang.liu@rutgers.edu (X. Liu).

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It is proved that the mechanism has the properties of “individual rationality” and “incentive compatibility”, respectively indicating that passengers’ actual paid prices will never exceed their maximum willing-to-pay prices and that truthfully reporting the personalized requirements is passengers’ optimal strategy, if the mechanism is obtained by exact algorithms. The mechanism needs to solve one optimization problem to obtain the optimal vehicle-passenger matching and vehicle routing plan, as well as to solve n (the number of requests sent from passengers) different optimization models for calculating n prices for all passenger requests. All optimization models in the mechanism are extensions of the vehicle routing problem and thus are NP hard (Lenstra and Kan, 1981), which cannot be solved exactly within polynomial time. Thus, obtaining the desired mechanism has to address highly challenging computational complexity. Previous studies on truth-inducing mechanisms (Kamar and Horvitz, 2009; Cheng et al., 2014; Zhao et al., 2014; Zhao et al., 2015; Asghari et al., 2016; Asghari and Shahabi, 2017; Shen et al., 2016; Nguyen, 2013; Zhang et al., 2016; Kleiner et al., 2011; Lloret-Batlle et al., 2017; Masoud et al., 2017b; Masoud and Lloret-Batlle, 2016; Ma et al., 2018) for ridesharing organization have not developed effective solution algorithms that can handle large-scale, complex, NP-hard, mechanism design models (particularly Vickrey-Clarke-Groves prices, VCG, Vickrey, 1961; Clarke, 1971; Groves, 1973). Thus, this paper aims at addressing the challenging computational issue of mechanism obtainment.

When the scale of the problem is large, approximation or heuristic algorithms are more applicable to obtain the mechanism. However, VCG-based mechanisms obtained by regular approximation or heuristic algorithms may no longer be able to sustain the game theoretic properties of “individual rationality” and “incentive compatibility” (Nisan and Ronen, 2007); our mechanism is no exception. Several researchers developed some special approximation or heuristic algorithms to maintain “individual rationality” and/or “incentive compatibility” in obtaining their mechanisms. For example, Lehmann et al. (2002) proposed an approximately efficient mechanism for combinatorial auctions using a greedy algorithm; Mu’Alem and Nisan (2008), Parkes and Ungar (2001) and Dobzinski et al. (2010) developed approximation mechanisms that are incentive compatible for combinatorial auctions; Nisan and Ronen (2007) proposed a second chance mechanism to circumvent the problem, upon which participants can do no better than be truthful. Nevertheless, all of the methods are designed specifically for combinatorial auctions. These algorithms have never been adapted to solve generalized mechanism design models.

Based on the discussion of the knowledge gap, this paper contributes to addressing these challenges by developing a computationally efficient heuristic algorithm called the Solution Pooling Approach (SPA). The application of the SPA is not limited to the mechanism design problem for first-mile ridesharing, but also can be spread to solve general mechanism design problems. Firstly, this paper introduces the basic idea of the SPA to solve generalized mechanism design problems, and analyzes specific circumstances under which the SPA is able to sustain game-theoretic properties, including “individual rationality” and “incentive compatibility”. The limitations of the SPA are identified: if the SPA needs to sustain “incentive compatibility”, it may sacrifice solution quality more significantly than traditional heuristic algorithms compared with exact algorithms. Then, this paper designs a specific SPA to obtain the personalized-requirement-based mechanism for the scheduled first-mile ridesharing service. We prove that the mechanism obtained by the SPA is still “individual rational” and “incentive compatible”. Moreover, SPA can reduce the computational time by simultaneously handling all models in this specific mechanism and does not need to solve all NP-hard problems one by one to obtain the mechanism. Numerical examples show that the SPA is more efficient than commercial solvers (e.g. ANTIGONE) and the conventional heuristic algorithms (e.g. Hybrid Simulated Annealing–Tabu Search Algorithm and Hybrid Genetic Algorithm) with a minor sacrifice of solution quality.

This paper is structured as follows. Section 2 briefly introduces the basic idea of the SPA to solve generalized mechanism design problems. Section 3 applies the SPA algorithm to solve the mechanism design problem for first-mile ridesharing based on passengers’ personalized requirements. In Section 4, numerical examples are provided to verify the effectiveness of the SPA. Concluding remarks are made in Section 5.

2. Basic idea of SPA to solve generalized mechanism design problems

This section proposes the generalized Solution Pooling Approach (SPA) to obtain results of generalized mechanisms for large-scale complex problems. This section also presents the basic idea of the SPA as being able to sustain game-theoretic properties, “individual rationality” and “incentive compatibility”, indicating that the SPA is not limited to the mechanism for the first-mile ridesharing but can also be adapted to solve other mechanism problems. The detailed design of the SPA for the specific application in the first-mile ridesharing is presented in Section 3.

2.1. Generalized mechanism design problems

The generalized mechanism design problem can be described by Fig. 1. The market maker wants to design a mechanism to incentivize participants’ collaboration to achieve a desirable objective (e.g. minimizing cost and maximizing the social welfare). Participants are allowed to report their personalized information to the system. Let $\theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$ denote all participants’ reported information. Based on participants’ reported information, the mechanism needs to determine a plan ($X = O(\theta)$) and an incentive function (I_i). The plan (e.g. resource allocating plan, vehicle routing plan, matching plan, etc.) aims to achieve the desirable objective usually by solving an optimization model. We denote this optimization model as *IP*. Then the market maker will design an incentive function, which is denoted as $p_i = I_i(X, \theta)$, for individuals’ participation

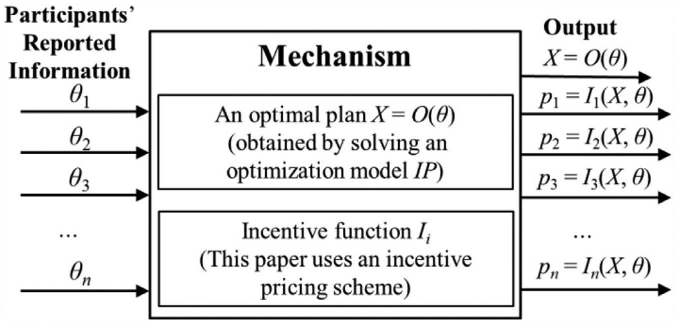


Fig. 1. Generalized mechanism design (Mishra, 2008).

based on the plan and participants' reported information. The incentive function has various forms, such as discounts, bonus points, credits, free service, etc. This paper typically uses customized pricing as an incentive form.

2.2. A generalized individual rational and incentive compatible mechanism

In order to achieve the market maker's objective, the mechanism should 1) ensure that the participants are willing to collaborate with each other and 2) prevent them from manipulating the mechanism by intentionally misreporting their personalized information. These two considerations necessitate the properties of "individual rationality" and "incentive compatibility". "Individual rationality" indicates that the actual paid prices will never exceed participants' maximum willing-to-pay prices. "Incentive compatibility" requires that participants' utilities (defined as the difference between the maximum willing-to-pay price and the actual paid price) can be maximized if they truthfully report their personalized information. This section proposes a generalized individual rational and incentive compatible mechanism. The optimal plan is obtained by solving the model IP. The pricing framework is calculated by designing and solving a series of models, including one model IP_0 and n models IP_g (corresponding to participant g). Model IP_0 should be equivalent to the original optimization model IP and thus the optimal solutions of models IP and IP_0 are identical. Models IP_g are used to calculate the prices only and do not have practical meaning. Both models IP_0 and IP_g use participants' reported information (θ) as input data and both have maximizing objective functions. Then the pricing scheme is given by

$$p_g = g(X^{IP_g*}) - (f(X^{IP_0*}) - V_g(X^{IP_0*})) \tag{1}$$

p_g is participant g's price. X^{IP_g*} is the optimal solution of model IP_g, and $g(.)$ is the maximizing objective function of the model. X^{IP_0*} is the optimal solution of model IP_0 with the maximizing objective function $f(.)$. $V_g(X)$ is participant g's value, which is defined as participant g's maximum willing-to-pay price in this paper, given the plan X. The objective function $f(.)$ includes summation of all participants' values.

$$f(X) = \sum_{i \in P} V_i(X) + h(X) \tag{2}$$

where $h(X)$ is used to make the model IP_0 equivalent to the original model IP.

This pricing scheme makes the mechanism "individual rational" if the following condition is always satisfied

$$g(X^{IP_g*}) \le f(X^{IP_0*}) \tag{3}$$

This is because participant g's utility is $U_g = V_g(X^{IP_0*}) - p_g = f(X^{IP_0*}) - g(X^{IP_g*}) \ge 0$, if the condition above is satisfied. A direct way of satisfying this condition is to design the model IP_g that makes the objective function $g(X)$ identical with $f(X)$ and to let the feasible regions of models IP_g (for each g) be included in the feasible region of model IP_0. That is

$$g(X) = f(X) \tag{4}$$

$$CS_{IP_g} \subseteq CS_{IP_0} \tag{5}$$

where CS_{IP_g} and CS_{IP_0} are the feasible regions of models IP_g and IP_0, respectively.

If model IP_g is independent of participant g's report, then the mechanism is "incentive compatible".

If participant g misreports her personalized information, then we assume that the optimal solution of model IP_0 changes from X^{IP_0*} to Y^{IP_0*}, $g(X^{IP_g*})$ remains constant because $g(X^{IP_g*})$ is independent of participant g's report, and $f(X^{IP_0*})$ changes to

$$f'(Y^{IP_0*}) = \sum_{i \in P, i \neq g} V_i(Y^{IP_0*}) + V'_g(Y^{IP_0*}) + h(Y^{IP_0*})$$

Then, the price becomes

$$p'_g = g(X^{IP_g*}) - \left(f'(Y^{IP_0*}) - V'_g(Y^{IP_0*})\right) = g(X^{IP_g*}) - \left(\sum_{i\in P, i\neq g} V_i(Y^{IP_0*}) + h(Y^{IP_0*})\right)$$

Then participant g's utility becomes

$$U'_g = V_g(Y^{IP_0*}) - p'_g = \left(\sum_{i\in P} V_i(Y^{IP_0*}) + h(Y^{IP_0*})\right) - g(X^{IP_g*}) = f(Y^{IP_0*}) - g(X^{IP_g*})$$

Y^{IP_0*} may no longer be optimal for model IP_0, indicating that the objective function of model IP_0, $f(.)$, will suffer from a decrease caused by her misreporting. Thus, her utility $U_g = f(X^{IP_0*}) \ - g(X^{IP_g*})$ may decrease as well if she misreports her personalized information. Therefore, truthful reporting is participants' optimal strategy:

$$U'_g = f(Y^{IP_0*}) - g(X^{IP_g*}) \leq f(X^{IP_0*}) - g(X^{IP_g*}) = U_g \tag{6}$$

The famous Vickrey-Clarke-Groves (Vickrey, 1961; Clarke, 1971; Groves, 1973) mechanism, which is widely applied in various research fields (Friedman and Parkes, 2003; Kamar and Horvitz, 2009; Samadi et al., 2012, etc.), belongs to this category and thus has the properties of "individual rationality" and "incentive compatibility".

2.3. SPA to the individual rational and incentive compatible mechanism

If the optimization models in the mechanism are NP hard, they are difficult to be solved exactly within a reasonable time when the problem scale is large. Many researchers (Wang et al., 2016; Lin et al., 2016; Gupta et al., 2017; Chao et al., 2017, etc.) have sought heuristic or approximation algorithms to find a high-quality solution to their optimization problems instead of an exact one. However, applying traditional heuristic or approximation algorithms may lose the properties of "individual rationality" and "incentive compatibility". Let us return to the generalized mechanism in Section 2.2. The mechanism is "individual rational" if the condition $g(X^{IP_g{}^*}) \leq f(X^{IP_0*})$ (Formula (3)) is always satisfied. However, if the solution X^{IP_0*} is obtained by a heuristic or approximation algorithm, the optimality of X^{IP_0*} cannot necessarily be guaranteed, and thus it is possible that $f(X^{IP_0*}) \ \leq g(X^{IP_g*})$ and $U_g < 0$. The property "individual rationality" is thus possibly violated. Similarly, the mechanism obtained by heuristic or approximation algorithms may not be incentive compatible as well. Suppose that X^{IP_0*} is the optimal solution of model IP_0 if participant g truthfully reports his personalized information and the solution becomes Y^{IP_0*} (not necessarily optimal) if participant g misreports the information. If heuristic or approximation algorithms are used to solve the model, it is possible that $f(Y^{IP_0*}) > f(X^{IP_0*})$, because the optimality of X^{IP_0*} cannot be guaranteed. Thus, implied from Formula (6), participants' utilities may not be maximized even though they tell the truth. Similar conclusions have already been drawn by other researchers (Mu'Alem and Nisan, 2008; Parkes and Ungar, 2001; Dobzinski et al., 2010; Nisan and Ronen, 2007).

Therefore, this paper proposes a special heuristic algorithm, namely the Solution Pooling Approach (SPA), to obtain the mechanism, sustaining the properties of "individual rationality" and "incentive compatibility" under specific circumstances. The SPA was inspired by the work of Bent and Hentenryck's (2004) multiple plan approach and Gendreau et al.'s (1999) tabu search algorithm organized around multiple solutions and an adaptive memory. The basic idea of the SPA can be described as follows. Firstly, the algorithm generates high-quality solutions of models IP_0 and IP_g (for all participants g) as solution pools. Then, the solutions of corresponding models with the highest qualities are selected from the solution pools. Let $Xpool^{IP_0}$ and $Xpool^{IP_g}$ denote the solution pools of model IP_0 and IP_g, respectively. Let X^{IP_0*} and X^{IP_g*} denote the optimal solutions in the pools $Xpool^{IP_0}$ and $Xpool^{IP_g}$, respectively. Then, X^{IP_0*} is adopted as the matching and routing plan, and the pricing scheme still adopts Formula (1).

When generating solution pools, if the condition $g(X^{IP_g*}) \leq f(X^{IP_0*})$ (Formula (3)) is still satisfied, the mechanism is "individual rational". If we use Formulas (4) and (5) to satisfy Formula (3), the SPA can easily guarantee "individual rationality". Since the feasible regions of models IP_g (for all g) are included in the feasible region of model IP_0, feasible solutions of model IP_g are feasible for model IP_0 as well. Solutions in pools $Xpool^{IP_g}$ can be integrated into the solution pool $Xpool^{IP_0}$. After the algorithm generates all of the solution pools $Xpool^{IP_g}$, all solutions in each pool $Xpool^{IP_g}$ are combined into the solution pool $Xpool^{IP_0}$ (i.e. $Xpool^{IP_g} \subset Xpool^{IP_0}$, for any g). Then, the optimal solution (X^{IP_0*}) is selected from $Xpool^{IP_0}$, and each X^{IP_g*} is selected from $Xpool^{IP_g}$. Since we have $X^{IP_g*} \in Xpool^{IP_g} \subset Xpool^{IP_0}$ and X^{IP_0*} is the optimal solution in $Xpool^{IP_0}$ with maximized objective $f(.)$, $f(X^{IP_g*}) \leq f(X^{IP_0*})$. Based on Formula (4), $g(X^{IP_g*}) = f(X^{IP_g*}) \leq f(X^{IP_0*})$, and then "individual rationality" can be guaranteed.

The property "incentive compatibility" is naturally guaranteed as long as the generation of $Xpool^{IP_g}$ (for each g) is independent of participant g's report and the $Xpool^{IP_0}$ is pre-generated before participants' personalized information is revealed. If each $Xpool^{IP_g}$ is independent of participant g's report, $g(X^{IP_g*})$ in Formula (6) remains constant regardless of participant g's report. Moreover, both X^{IP_0*} and Y^{IP_0*} in Formula (6) are selected from the pre-generated pool $Xpool^{IP_0}$. Since X^{IP_0*} is the optimal solution in $Xpool^{IP_0}$ while Y^{IP_0*} is not necessarily the optimal in $Xpool^{IP_0}$, $f(Y^{IP_0*}) \leq f(X^{IP_0*})$ in Formula (6) can always be satisfied. Thus, the mechanism obtained by the SPA is incentive compatible.

The SPA is an efficient heuristic algorithm that can sustain the properties of "individual rationality" and "incentive compatibility" under the specific circumstances analyzed above, but it still has limitations. The SPA needs to pre-generate the

Table 1
Mathematical models for obtainment of the mechanism.

Model denotations	Objective functions	Constraints	Optimal solution	Optimal objective function value
IP_0	$f(X)$: Formula (A2) Max $Z_0(X)$	CS_{IP_0} Formulas (A3)–(A12)	X^{IP_0*}	$Z^*_{IP_0}$
IP_g for all $g \in P$	$g(X)$: Formula (A2) Max $Z_0(X)$	CS_{IP_g} Formulas (A3)–(A13)	X^{IP_g*}	$Z^*_{IP_g}$

solution pools for models IP_0 and IP_g before participants report their personalized information in order to sustain "incentive compatibility", leading to a potential sacrifice of solution quality. The SPA has to significantly increase the number of solutions in the pool in order to improve the solution quality, but this will consume more computer memory. In Section 4, our numerical examples are designed to test how much the SPA will sacrifice the solution quality in obtaining the mechanism for first-mile ridesharing service.

3. Application of SPA to solve the mechanism design problem for first-mile ridesharing

This section details the SPA to the specific mechanism for the scheduled first-mile ridesharing service proposed in our Part I paper.

3.1. Mechanism design problem for first-mile ridesharing based on personalized requirements

This subsection reviews the personalized-requirement-based mechanism design problem for a first-mile ridesharing service. Passengers near the transit hub book the first-mile ridesharing service in advance. The service provider dispatches a fleet of vehicles to execute the pickup and drop-off tasks. Each request specifies a deadline before which passenger(s) must arrive at the transit hub. In addition to the passengers' pickup locations and the arrival deadlines, passengers are allowed to report their personalized requirements on three inconvenience factors: the number of co-riders, extra in-vehicle travel time, and extra waiting time at the transit hub. Before vehicles are dispatched, the system will determine an optimal vehicle-passenger matching and vehicle routing plan X^* and all passengers' customized prices $\mathbf{p} = \{p_1, p_2, \ldots, p_n\}$, which form the mechanism $M(X^*, \mathbf{p})$. The optimal matching and routing plan X^* is obtained by solving an optimization model (denoted as IP) with the objective of minimizing passengers' inconvenience cost and the service provider's transportation cost. The model IP is the formulated by Formulas (A.1, A.3–A.12) in Appendix A. The pricing scheme is obtained by solving a series of optimization models, including one model IP_0 and n models IP_g for all $g \in P$, which are summarized in Table 1. For the notations and formulas, please refer to Appendix A.

Note that model IP_0 is equivalent to model IP, and thus the optimal solutions of models IP_0 and IP are identical ($X^{IP_0*} = X^*$). The mechanism is denoted as $M(X^{IP_0*}, \mathbf{p})$, where X^{IP_0*} represents the optimal vehicle-passenger matching and routing plan, and passengers' prices $\mathbf{p} = \{p_1, p_2,..., p_n\}$ are calculated by

$$p_g = Z^*_{IP_g} - \left(Z^*_{IP_0} - V_g\left(X^{IP_0*}\right)\right) \tag{7}$$

Note that models IP_0 and IP_g have identical objective functions, and that the feasible region of model IP_g is included in the feasible region of model IP_0 because model IP_g has an additional constraint (Formula A.13) compared with model IP_0. Thus, the mechanism is "individual rational" based on Formulas (4) and (5). Moreover, the optimal solution is independent of passenger(s) g's report of the parameters of ${\alpha_g}^{NR}$, ${\alpha_g}^{IVT}$, and ${\alpha_g}^{WT}$ (see notations in Appendix A) because passenger(s) g's inconvenience cost is zero and the value is a constant ($V^g_{\max}$) if the passenger(s) is transported to the transit hub directly without shared riders, no matter what values of ${\alpha_g}^{NR}$, ${\alpha_g}^{IVT}$, and ${\alpha_g}^{WT}$ the passenger(s) reports. Thus, the mechanism is "incentive compatible". For detailed proof of these two properties, please refer to our Part I paper.

3.2. Identified challenges to obtain the mechanism

The optimization models in the mechanism, including IP_0 and IP_g ($g \in P$), are extensions of the classical vehicle routing problem and thus are NP hard (Lenstra and Kan, 1981). When the scale of the problem is large, exact algorithms face difficulty in obtaining the optimal solution within a reasonable time. Heuristic algorithms are more applicable for large-scale problems. When passengers send n requests, the mechanism $M(X^{IP_0*}, \mathbf{p})$ includes $n+1$ NP hard optimization models, including one optimization model IP_0 used to determine the optimal vehicle-passenger matching and vehicle routing plan X^{IP_0*} and n optimization models IP_g ($g = 1, 2, \ldots, n$) that are used to calculate all prices. Regular heuristic algorithms (e.g. Simulated Annealing and Genetic Algorithm) are still time-consuming in solving these models one by one. Moreover, as analyzed in Section 2.3, if traditional heuristic algorithms are used to obtain the mechanism $M(X^{IP_0*}, \mathbf{p})$, the properties of "individual rationality" and "incentive compatibility" are not necessarily guaranteed. To overcome these challenges, we implement the SPA to obtain the mechanism $M(X^{IP_0*}, \mathbf{p})$.

3.3. SPA to the first-mile ridesharing mechanism

The generation of $Xpool^{IP_0}$ and $Xpool^{IP_g}$ can be described as follows. First, the SPA generates an initial solution pool in which all solutions are feasible for the optimization model IP_0 (see Algorithm 1). We denote it as *Xpool*. Then solution pools of models IP_g ($g = 1, 2, \ldots, n$) are obtained based on *Xpool*. We design a transition solution generation algorithm (see Algorithm 2) to generate solution pools $Xpool^{IP_g}$ of models IP_g. All solutions in $Xpool^{IP_g}$ for all $g = 1, 2, \ldots, n$ are combined into the initial pool *Xpool*, and a new solution pool $Xpool^{IP_0}$ of model IP_0 is generated. Finally, the optimal solutions are selected from corresponding solution pools $Xpool^{IP_0}$ and $Xpool^{IP_g}$. The matching and routing plan adopts the optimal solution selected from the pool $Xpool^{IP_0}$. All passengers' prices are calculated based on Formula (7).

The initial solution pool *Xpool* should be pre-generated and passengers' reported personalized requirements (${\alpha_i}^{NR}$, ${\alpha_i}^{IVT}$, and ${\alpha_i}^{WT}$) on the three inconvenience attributes do not influence the generation of the solution pool so that the mechanism obtained by the SPA is still incentive compatible (please refer to the proof of the incentive compatibility proposition of the SPA in Section 3.4). Thus, we propose two strategies to improve the quality of the solution selected from the obtained solution pool of IP_0: 1) generate a large enough number of solutions in the solution pool *Xpool* and select the best solutions from the pool; 2) randomly and periodically simulate virtual personalized requirements parameters (${\alpha_i}^{NR}$, ${\alpha_i}^{IVT}$, and ${\alpha_i}^{WT}$) that are used to direct the generation of solutions in a wide range and thereby guarantee the quality of the optimal solution. A meta-heuristic algorithm, tabu search (TS) plays the role of solution generator. TS is able to avoid repeated generation of identical solutions using a memory function (Gendreau et al., 1994). Algorithm 1 gives the pseudocode of the solution pool generation algorithm.

Then we introduce the definition of the "transition solution", which is also defined in our Part I paper. The "transition solution" will be used to obtain solution pools $Xpool^{IP_g}$ of model IP_g for all $g = 1, 2, \ldots, n$ in the SPA algorithm. Let $Y_g = TRS_g(X)$ be the g^{th} transition solution from a feasible solution X of the model IP_0. The transition process is generated as follows. Let passenger(s) g go to the transit hub directly without any other shared riders, and let the broken routes re-connect. In Y_g, since passenger(s) g is transported to the transit hub directly without shared riders, Formula (A.13) is satisfied and Y_g is a feasible solution of model IP_g.

Fig. 2 shows an example of transition solution generation. Algorithm 2 shows how to get the transition solution.

Algorithm 2 is used to get the gth transition solution Y_i^g of each $X_i \in Xpool$: $Y_i^g = TRS_g(X_i)$. Since Y_i^g (for all i) are all feasible to model IP_g, the solution pool $Xpool^{IP_g}$ consists of all Y_i^g.

Algorithm 1 Generation of solution pool *Xpool*.

```
Input the total number of iterations (NI), number of iterations in each period (NIP)
for updating values α_i^NR, α_i^IVT and α_i^WT, number of candidate solutions (CN),
number of solutions (NS) assigned into the solution pool for each iteration, and all
other parameters of the problem;
Initialize a feasible solution X_0 to the model IP_0 as the current solution X_current, the
virtual values of α_i^NR, α_i^IVT and α_i^WT, it = 0 (current number of iterations), pit = 0
(current number of iterations in one period), and the empty solution pool Xpool;
Do while it < NI
    If  pit > NIP
        pit = 0;
        Use the uniform distribution to re-generate the values of α_i^NR, α_i^IVT, and
        α_i^WT;
    End if
    Generate CN candidate solutions {X_1, X_2,…, X_CN} of X_current's neighbors;
    Calculate {ΔZ_0(X_1), ΔZ_0(X_2),…, ΔZ_0(X_CN)} (ΔZ_0(X_i) = Z_0 (X_i) − Z_0 (X_current)) and
    record the subscript opt, where ΔZ_0(X_opt) = max{ΔZ_0(X_1), ΔZ_0(X_2), … ,
    ΔZ_0(X_CN)};
    Randomly select NS solutions from CN candidate solutions {X_1, X_2,…, X_CN}
    and put them into the solution pool Xpool;
    Do while X_opt is in tabu list
        Select the suboptimal solution as X_opt from {X_1, X_2, …, X_CN};
    End do
    X_current = X_opt;
    Update the tabu list;
    it = it + 1;
    pit = pit + 1;
End do
Output Xpool.
```

Algorithm 2 Obtain the transition solutions $Y_g = TRS_g(X)$ and calculate the objective function $Z_0(Y_g)$.

Input a solution $X = \{x_{ijk}, y_{ik}, w_{ik}\}$ and the objective function value $Z_0(X)$;
Let $Y_g = X$;
If $NR_g > 0$
    Find k that $y_{gk} = 1$, and let $y_{gk} = 0$;
    $Z_0^k(X) = V^k(X) - TC^k(X)$;
      % $V^k(X)$: summation of values of passengers in vehicle k given plan X
        $TC^k(X)$: vehicle k's transportation cost given plan X.
    $y_{gk'} = 1$, $w_{gk'} = 1$, and $x_{g0k'} = 1$;
      % Let another vehicle k' without tasks pick up passenger(s) g.
    Find j that $x_{gjk} = 1$, and let $x_{gjk} = 0$;
    If $w_{gk} = 0$
        Find i that $x_{igk} = 1$, and let $x_{igk} = 0$;
        Let $x_{ijk} = 1$;
    Else
        Let $w_{gk} = 0$;
        Let $w_{jk} = 1$;
    End if
    $Z_0^k(Y_g) = V^k(Y_g) - TC^k(Y_g)$;
    $Z_0(Y_g) = Z_0(X) - Z_0^k(X) + Z_0^k(Y_g) + V_{\max}^g - c_{g0}$;
Else
    $Z_0(Y_g) = Z_0(X)$;
End if
Output Y_g and $Z_0(Y_g)$.

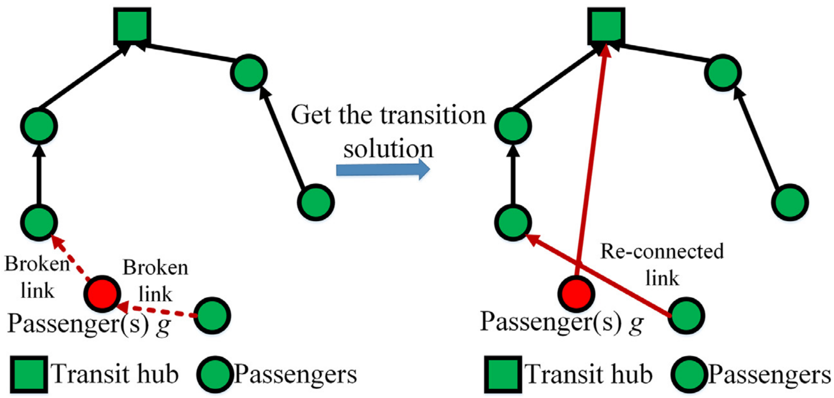


Fig. 2. Example of the transition solution.

Algorithm 3 is then developed to get the mechanism, including the optimal matching and routing plan and all passengers' prices. Fig. 3 is the flow chart of the SPA, which indicates that the SPA can simultaneously handle all NP-hard models in the mechanism and does not need to solve them one by one.

3.4. Theoretical analysis of SPA

The propositions of this mechanism, including "individual rationality" and "incentive compatibility", are still valid if the SPA is used to obtain the mechanism$M(X^{IP_0*}, \mathbf{p})$. Before giving the proof of the propositions, we re-formulate the problems based on the SPA.

In Algorithm 3, X^{IP_0*} is the optimal solution selected from the solution pool $Xpool^{IP_0}$, and thus X^{IP_0*} is also the optimal solution of the optimization model below (Formulas (8) and (9)). We denote this model as $IPpool_0$.

$$Z^*_{IP_0} = \max Z_0(X) \tag{8}$$

Subject to,

$$X \in Xpool^{IP_0} \tag{9}$$

Similarly, X^{IP_g*} is the optimal solution of the optimization model below, which is used for calculation of passenger(s) g's customized price. We denote this model as $IPpool_g$ (Formulas (10) and (11))

$$Z^*_{IP_g} = \max Z_0(X) \tag{10}$$

Subject to,

$$X \in Xpool^{IP_g} \tag{11}$$

Proposition 1. Individual Rationality

If X^{IP_0*} and X^{IP_g*} are the optimal solutions of $IPpool_0$ and $IPpool_g$, respectively, the mechanism $M(X^{IP_0*}, \mathbf{p})$ is individual rational, i.e. the utility

$$U_g\left(X^{IP_0*}, p_g\right) = V_g\left(X^{IP_0*}\right) - p_g \geq 0, \text{for any } g \in P \tag{12}$$

Algorithm 3 SPA to the mechanism.

Input $Xpool$ obtained by Algorithm 1 and all parameters of the problem;
Calculate $Z_0(X_i)$ for all $X_i \in Xpool$;
For $g = 1{:}n$
  Use Algorithm 2 to get the transition solutions of all the solutions in $Xpool$ as the solution pool of IP_g, $Xpool^{IP_g}$:
  $Xpool^{IP_g} = \left\{Y_i^g \middle| Y_i^g = TRS_g\left(X_i\right), \text{ for all } X_i \in Xpool\right\}$;
  $\mathbf{Z}_g = \left\{Z_0\left(Y_i^g\right), \text{for all } Y_i^g \in Xpool^{IP_g}\right\}$;
End for
Let $Xpool^{IP_0} = \left\{Xpool, Xpool^{IP_g} (\text{for all } g \in P)\right\}$;
Select the solution X^{IP_0*} from $Xpool^{IP_0}$ that $X^{IP_0*} = \arg\max Z_0\left(X\right)$
$X \in Xpool^{IP_0}$;
Let $Z^*_{IP_0} = Z_0\left(X^{IP_0*}\right)$;
For $g = 1{:}n$
  Select the solution X^{IP_g*} from $Xpool^{IP_g}$ that $X^{IP_g*} = \arg\max Z_0\left(Y_i^g\right)$
  $Z_0\left(Y_i^g\right) \in \mathbf{Z}_g$;
  Let $Z^*_{IP_g} = Z_0\left(X^{IP_g*}\right)$;
  Calculate the prices:
  $p_g = Z^*_{IP_g} - \left(Z^*_{IP_0} - V_g\left(X^{IP_0*}\right)\right)$;
End for
Output the optimal solution of IP_0 (X^{IP_0*}) and all passengers' prices
$\mathbf{p} = \left\{p_1, p_2, \ldots, p_n\right\}$.

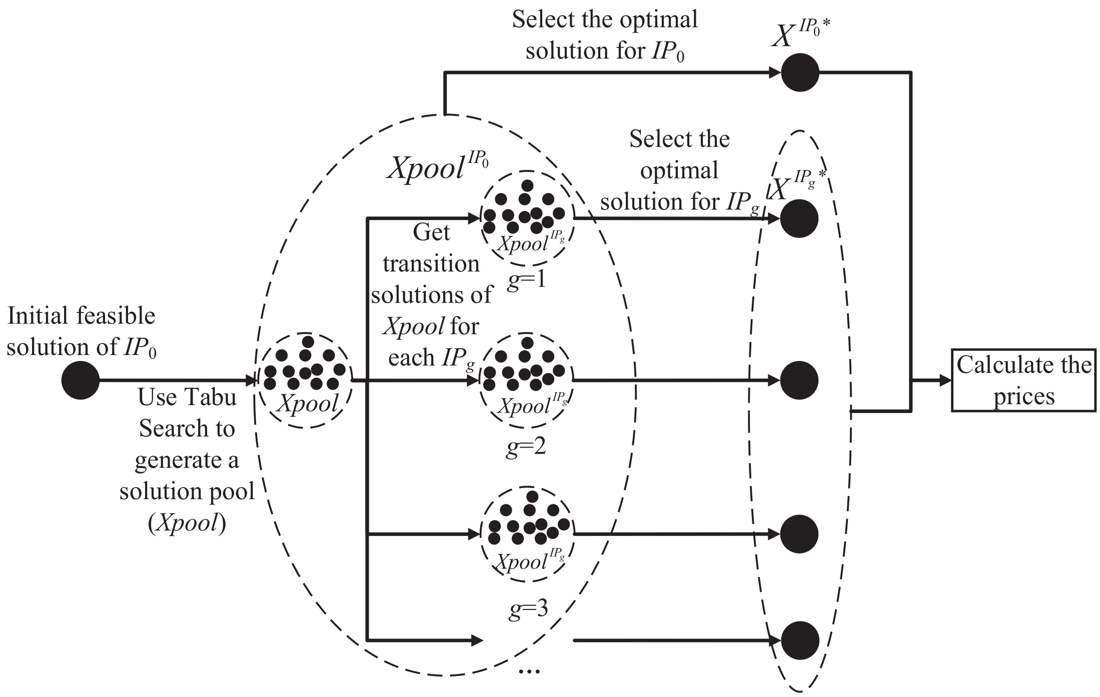


Fig. 3. Flow chart of SPA in obtaining the mechanism.

Proof. X^{IP_0*} is the optimal solution of the model $IPpool_0$, and thus $Z_0(X^{IP_0*}) \geq Z_0(X)$ for any $X \in Xpool^{IP_0}$. Since $X^{IP_g*} \in Xpool^{IP_g} \subseteq Xpool^{IP_0}$, X^{IP_g*} is a feasible solution of $IPpool_0$. Thus $Z_0(X^{IP_0*}) \geq Z_0(X^{IP_g*})$.

$$\begin{aligned} & U_g\left(X^{IP_0*}, p_g\right) = V_g\left(X^{IP_0*}\right) - p_g \\ & = V_g\left(X^{IP_0*}\right) - Z_0\left(X^{IP_g*}\right) + \left(Z_0\left(X^{IP_0*}\right) - V_g\left(X^{IP_0*}\right)\right) \\ & = Z\left(X^{IP_0*}\right) - Z_0\left(X^{IP_g*}\right) \geq 0 \end{aligned}$$

This suggests that the participants can always receive non-negative utility from the first-mile ridesharing service. □

Proposition 2. Incentive Compatibility

The mechanism $M(X^{IP_0*}, \mathbf{p})$ is incentive compatible if the optimal matching and routing plan X^{IP_0*} and the prices $\mathbf{p}$ are obtained by the SPA.

Proof. We assume that passenger(s) g misreports the parameters (personalized requirements, denoted as θ_g) in the value function.

We define $V_g{}'(X) = V_{\max}^g - C^{ICN}(NR_g(X), IVT_g(X), WT_g(X), \theta_g')$regardless of other passengers' reporting strategies, where θ_g' is the set of passenger(s) g's misreported values in θ_g.

Since $Xpool^{IP_0}$ and $Xpool^{IP_g}$ are all generated independently on all passengers' reports of their personalized requirements, the solution pools $Xpool^{IP_0}$ and $Xpool^{IP_g}$ remain constant no matter how passengers report their requirements. Thus, the constraints of $IPpool_0$ and $IPpool_g$ remain constant regardless of passengers' reporting strategies. If passenger(s) g misreports the requirements, the optimization model $IPpool_0$ becomes $IPpool_0{}'$:

$$Z_{IP_0{}'}^* = \max Z_0{}'(X) = \sum_{i \in P, i \neq g} V_i(X) + V_g{}'(X) - TC(X),\ s.t. X \in Xpool^{IP_0}$$

Other passengers' values are calculated based on their actual report of personalized requirements regardless of the truthfulness. The optimal solution (denoted by $X^{IP_0{}'*}$) of $IPpool_0{}'$ is still feasible for $IPpool_0$. We have $Z_0(X^{IP_0*}) \geq Z_0(X^{IP_0{}'*})$ because X^{IP_0*} is the optimal solution of $IPpool_0$ and $X^{IP_0{}'*}$ is a feasible solution of $IPpool_0$. Moreover, the model $IPpool_g$ never changes, no matter what passenger(s) g reports. This is because 1) the constraints of $IPpool_g$ remains constant no matter how passengers report their requirements and 2) the objective function value is independent of passenger(s) g's report. Thus passenger(s) g's price is

$$p_g{}' = Z_{IP_g}^* - \left(Z_{IP_0{}'}^* - V_g{}'\left(X^{IP_0{}'*}\right)\right)$$

The utility that passenger(s) g can receive is:

$$
\begin{aligned}
&U_g\left(X^{IP_0{}'*}, p_g{}'\right) = V_g\left(X^{IP_0{}'*}\right) - p_g{}' \\
&= V_g\left(X^{IP_0{}'*}\right) - \left(Z^*_{IP_g} - \left(Z^*_{IP_0{}'} - V_g{}'\left(X^{IP_0{}'*}\right)\right)\right) \\
&= V_g\left(X^{IP_0{}'*}\right) - \left(Z^*_{IP_g} - \left(\sum_{i\in P, i\neq g} V_i\left(X^{IP_0{}'*}\right) + V_g{}'\left(X^{IP_0{}'*}\right) - TC\left(X^{IP_0{}'*}\right) - V_g{}'\left(X^{IP_0{}'*}\right)\right)\right) \\
&= \sum_{i\in P} V_i\left(X^{IP_0{}'*}\right) - TC\left(X^{IP_0{}'*}\right) - Z^*_{IP_g} \\
&= Z_0\left(X^{IP_0{}'*}\right) - Z^*_{IP_g} \\
&\leq Z_0\left(X^{IP_0*}\right) - Z^*_{IP_g} \\
&= U_g\left(X^{IP_0*}, p_g\right)
\end{aligned}
$$

which indicates that the passenger receives the largest utility when telling the truth regardless of other passengers' reporting strategies. Therefore the mechanism is incentive compatible. □

Note that it is very difficult to develop approximate or heuristic algorithms to simultaneously guarantee "price non-negativity" as well as "individual rationality" and "incentive compatibility". The SPA algorithm is proven to be individual rational and incentive compatible, but it may not guarantee the property of "price non-negativity". However, the numerical experimental results in Section 4 show that the SPA never obtains negative prices.

4. Numerical experiment

4.1. Design of numerical examples

In our Part I paper, we developed a case study to interpret the results of the mechanism. However, it does not have generality because it only contains two specific scenarios, in which passengers have two different reporting methods and two types of value functions. Moreover, the scale of the problem in the case study is small: only ten passenger requests are involved. Thus it is not possible to test the effectiveness of the proposed algorithm in obtaining the large-scale generalized mechanism. In this paper, we develop thirteen numerical examples to test the proposed algorithm in obtaining the mechanism $M(X^{IP_0*}, \mathbf{p})$. In order to show the trend of experimental results with the scale of problems increasing, the number of passenger requests involved in the system increases from 4 to 52 by the interval of 4. Both horizontal and vertical coordinates (x_i, y_i) of all passenger locations in numerical examples are generated uniformly from the interval [6, 12]. All coordinates of the transit hubs are set to be (9, 9), approximately located in the center of all passengers. For convenience but without losing generality, the transportation cost between two locations is proportional to the Euclidean distance: $c_{ij} = 2d_{ij}$, where d_{ij} is the distance between two locations. We determine that $V^i_{\max} = 3 + 3d_{i0}$. The traveling time between two locations is not necessarily proportional to the distance. Thus, we use a different method to generate the travel time between two locations. Virtual coordinates (xv_i, yv_i) of locations are generated, which satisfy: $xv_i = x_i + \varepsilon$ and $yv_i = y_i + \varepsilon$. ε is normally distributed with the mean of "0" and variance of "0.1". ε is randomly generated by the computer. The travel time between i and j is set to be $t_{ij} = 3\sqrt{(xv_i - xv_j)^2 + (yv_i - yv_j)^2}$.

Passengers' personalized requirements (${\alpha_i}^{NR}$, ${\alpha_i}^{IVT}$, and ${\alpha_i}^{WT}$, in any form) on the three inconvenience attributes can be processed into an interval [0, 1], representing the strictness of the requirements. Since the passengers' personalized requirements are processed, we cannot use the value functions proposed in our Part I paper because the values of ${\alpha_i}^{NR}$, ${\alpha_i}^{IVT}$, and ${\alpha_i}^{WT}$ are no longer compatible with the value functions in the Part I paper. Thus, we propose another illustrative value function (Formula (13)), which is compatible with the processed values of ${\alpha_i}^{NR}$, ${\alpha_i}^{IVT}$, and ${\alpha_i}^{WT}$, and will be used in numerical examples to test the algorithm.

$$
V_i = V^i_{\max} - \left(\frac{\alpha_i^{NR} NR_i}{Q - np_i} + \frac{\alpha_i^{IVT}(IVT_i - t_{i0})}{t_{i0}} + \frac{\alpha_i^{WT} WT_i}{MD}\right)\left(\frac{V^i_{\max} - c_{i0}}{2}\right) \tag{13}
$$

MD is the maximum difference among passengers' arrival deadlines. Here $MD = \max_{i,j\in P}(DL_i - DL_j) = 15$, indicating that we only optimize the matching and routing plan connecting to train schedules in which differences in passengers' arrival deadlines do not exceed 15 min. We set the default values of ${\alpha_i}^{NR}$, ${\alpha_i}^{IVT}$, and ${\alpha_i}^{WT}$ to 0.1. In other words, if the passengers do not report their requirements, the system will adopt the default values. We set half of the values ${\alpha_i}^{NR}$, ${\alpha_i}^{IVT}$, and ${\alpha_i}^{WT}$ to 0.1 as the default values, indicating that half of passengers do not open the interface to place stricter personalized requirements for the ridesharing service. The other half of the values ${\alpha_i}^{NR}$, ${\alpha_i}^{IVT}$, and ${\alpha_i}^{WT}$ are randomly generated from the uniform distribution interval [0, 1]. Formula (13) builds on the assumption that passengers are willing to pay a price at least equal to the minimum transportation cost (c_{i0}) if they are transported to the transit hub within the minimum travel time t_{i0} ($V_i \geq c_{i0}$, if $IVT_i = t_{i0}$). This is a reasonable assumption because if passengers drive themselves to the transit hub, they have to bear the direct shipment cost (c_{i0}). Note that we use this hypothetical function just to test the algorithm and the accuracy of this value function has not been verified through practical survey. We will use another value function, in which passengers' attitude towards the price is stricter, in the sensitivity analysis to demonstrate the robustness of the proposed algorithm in obtaining the mechanism under different conditions.

4.2. Testing method and criteria

This subsection compares the solution pooling approach (SPA) with an exact algorithm, commercial solvers, and selected state-of-the-art heuristic algorithms. We use the enumeration algorithm (EA) as a representative of the exact algorithm to solve small-scale problems (numerical examples with 4 and 8 passenger requests). Effective exact algorithms (e.g. branch and bound) are not developed in this paper because they are difficult to adapt to generalized models with different objective functions. We use seven commercial solvers, ANTIGONE, ALPHAECP, BARON, COUENNE, LINDOGLOBAL, SBB, and SCIP (https://neos-server.org/neos/solvers/index.html), which are all able to solve mixed integer non-linear programming (MINLP) models (Bussieck and Vigerske, 2010) to obtain the mechanism results. For all the solvers, the maximum computing time in solving one MINLP model is set to 3600 s. Among the seven solvers, ANTIGONE has the highest performance both in terms of solution quality and computing speed. The possible reason is that ANTIGONE implements a spatial branch-and-bound algorithm that utilizes MIPs for bounding. The MIP relaxation is generated from a reformulation of the MINLP. It employs a large collection of convexification and bound tightening techniques (Bussieck and Vigerske, 2010). For conciseness, we select ANTIGONE to compare with the proposed SPA algorithm, but we attach the results of all seven solvers in Appendix B. Finally, it is difficult to test all state-of-the-art heuristic algorithms in the literature, but we select two representative heuristic algorithms for comparison with our proposed SPA. We select Hybrid Simulated Annealing - Tabu Search algorithm (HSATS) as a representative of local-search-based heuristic algorithms and select Hybrid Genetic and Local Search algorithm (HGLS) as a representative of swarm evolutionary heuristic algorithms. Both HSATS (Lin et al., 2016) and HGLS (Wang, 2014) are effective for solving the classic Traveling Salesman Problem (TSP). We modify the mutation structures (e.g. neighborhood structure and crossover structure) to adapt the algorithms to the first-mile ridesharing matching and routing problem. The algorithm comparison is based on the following criteria:

1) **Objective function values**. We compare the performances of EA, ANTIGONE, HSATS, HGLS, and SPA in terms of the objective function values of IP_0 for all numerical examples.
2) **Computing time**. Computing time is used to measure the efficiency of an algorithm. This paper will compare the computing time of ANTIGONE, HSATS, HGLS, and SPA in solving the optimization model IP_0 and calculating the prices.
3) **Mechanism properties**. We will show the reliabilities of these algorithms to sustain two properties “individual rationality” and “price non-negativity”. The property “incentive compatibility” is difficult to test and thus is not included in the comparison.
4) **Service provider profitability**. The experiment results will show if the price collected from passengers can cover the transportation cost.

4.3. Running conditions

The algorithms, EA, HSATS, HGLS, and SPA, are programmed in Matlab R2014a. The commercial solvers are implemented on the website of NEOS Solvers (https://neos-server.org/neos/solvers/index.html). All algorithms are implemented on a Dell computer with processor Intel(R) Core(TM) i7-4790 CPU @ 3.60 GHz and 8GB RAM.

4.4. Experiment results

We first compare five solution approaches, EA, ANTIGONE, HSATS, HGLS, and SPA, in terms of objective function values in solving model IP_0. Table 2 presents the comparison results. The numerical examples are denoted by “N_x”, where “x” is number of passenger requests.

EA is able to solve only two small-scale problems (N_4 and N_8). When the number of passenger requests reaches “12”, the computer registers a shortage of memory.

The solver ANTIGONE can return a solution, not necessarily optimal, within one hour (3600 s) for numerical examples with the numbers of passengers ranging from 4 to 28. The solution qualities obtained by ANTIGONE are very close to the heuristic algorithms, HGLS, HSATS, and SPA, in solving the numerical examples with passengers fewer than and equal to 24. When the number of passengers reaches “28”, the quality of the solution obtained by ANTIGONE is much lower than those obtained by the heuristic algorithms: the objective function value obtained by ANTIGONE is 159.28, much lower than 186.47 of HGLS, HSATS, and SPA. When the number of passengers is larger than 28, ANTIGONE is unable to return a solution.

All of the three heuristic algorithms HSATS, HGLS, and SPA are able to find solutions for all numerical examples. They obtain the exact optimal solutions of numerical examples N_4 and N_8 as EA does. With the scale of the problem increasing, the solution qualities of HSATS and HGLS are slightly higher than those of SPA in general. However, the differences between SPA and HSATS and between SPA and HGLS are negligible. The maximum difference between SPA and HSATS/HGLS is only 1.55% (N_36).

Table 3 shows the computing time for obtaining an optimal matching and routing plan and calculating prices, as well as the total computing time spent by ANTIGONE, HSATS, HGLS, and SPA. The commercial solver ANTIGONE is much more

Table 2
Objective function values obtained by EA, ANTIGONE, HSATS, HGLS, and SPA.

Numerical examples	EA	ANTIGONE	HSATS	HGLS	SPA		
					Objective function values	Difference from HSATS (%)	Difference from HGLS (%)
N_4	24.56	24.56	24.56	24.56	24.56	0.00	0.00
N_8	58.00	58.00	58.00	58.00	58.00	0.00	0.00
N_12		81.52	81.52	81.52	81.52	0.00	0.00
N_16		119.98	119.98	119.98	119.45	0.44	0.44
N_20		139.89	139.89	139.89	139.86	0.02	0.02
N_24		152.58	152.58	152.71	152.58	0.00	0.09
N_28		159.28	186.47	186.47	186.47	0.00	0.00
N_32			200.08	200.08	200.03	0.02	0.02
N_36			259.67	259.67	255.71	1.55	1.55
N_40			289.98	289.98	289.54	0.15	0.15
N_44			302.17	302.17	301.28	0.30	0.30
N_48			349.51	349.54	346.98	0.73	0.74
N_52			380.77	379.63	377.70	0.81	0.51

Note: the table only presents the data when the computer memory is sufficient and the computing time is less than or equal to one hour (3600 s).

Table 3
Computing time (in seconds) of ANTIGONE, HSATS, HGLS, and SPA.

Numerical examples	ANTIGONE			HSATS			HGLS			SPA		
	TO	TP	TT	TO	TP	TT	TO	TP	TT	TO	TP	TT
N_4	0.09	0.34	0.43	0.11	0.42	0.53	0.09	0.34	0.43	0.14	0.00	0.14
N_8	1.95	5.46	7.41	1.23	8.94	10.17	0.73	5.87	6.60	1.47	0.35	1.82
N_12	2278.09	>3600	>3600	3.70	42.47	46.17	2.61	31.34	33.95	4.79	1.24	6.03
N_16	3600.00	>3600	>3600	12.27	193.15	205.42	11.61	177.59	189.20	14.89	5.03	19.92
N_20	3600.00	>3600	>3600	16.86	298.28	315.14	20.57	380.14	400.71	19.93	7.15	27.08
N_24	3600.00	>3600	>3600	19.98	474.04	494.02	38.68	910.37	949.05	24.38	9.44	33.82
N_28	3600.00	>3600	>3600	26.82	694.69	721.51	66.40	1724.21	1790.61	35.32	12.22	47.54
N_32				32.02	877.13	909.15	86.35	2665.13	2751.48	39.30	18.82	58.12
N_36				32.33	1227.69	1260.02	101.23	3553.58	3654.81	46.83	25.43	72.26
N_40				39.94	1646.68	1686.62	118.06	>3600	>3600	56.70	31.78	88.48
N_44				45.18	2032.65	2077.83	141.86	>3600	>3600	66.88	37.94	104.82
N_48				53.85	2652.70	2706.55	173.49	>3600	>3600	71.00	43.98	114.98
N_52				59.37	3063.98	3123.35	189.85	>3600	>3600	89.96	59.70	149.66

Annotation: TO, computing time in obtaining the optimal routing plan; TP, computing time in calculating the prices; TT, the total computing time.

time-consuming than the three heuristic algorithms HSATS, HGLS, and SPA in getting the mechanism results for numerical examples with more than 12 passengers. HSATS needs more than 3000 s (50 min) to obtain the mechanism for the largest-scale numerical example (N_52), and HGLS is unable to obtain the mechanism for the largest-scale numerical example within one hour. In contrast, SPA is able to obtain the mechanism for all numerical examples within 3 min. This is because both HSATS and HGLS need to solve n similar optimization models one by one to calculate the prices given that the number of passenger requests is n, while SPA is able to solve these similar models simultaneously. Moreover, it can be inferred from Fig. 4 that the computing complexity of SPA is lower than those of HSATS and HGLS. With the scale of problems continuously increasing, the computing times of HSATS and HGLS increase faster than that of SPA.

The mechanism obtained by the exact algorithm was proven, in our Part I paper, to have three properties: "individual rationality", "incentive compatibility", and "price non-negativity". We compare the ability of the four algorithms (EA, HSATS, HGLS, and SPA) in maintaining these properties. ANTIGONE is not presented here because it is very time-consuming. Table 4 presents the percentages of individual rational and non-negative prices in the total number of prices using four algorithms for all numerical examples. If the properties "individual rationality" and "price non-negativity" are strictly proved, the table cell shows "proved". Otherwise, only a percentage is shown in the table. Table 4 shows that the mechanism obtained by both HSATS and HGLS are possibly not "individual rational". In numerical examples N_24 and N_48, at least one passenger's utility is negative in the mechanism obtained by HSATS (the bold numbers are less than 100%). In the numerical example N_32, at least one passenger's utility is negative in the mechanism obtained by HGLS. Negative utilities indicate that these passengers are unwilling to pay the prices. Our Part I paper and this paper respectively proved that the mechanisms obtained by EA and SPA are always individual rational, and thus all passengers' utilities are non-negative. Although we cannot strictly prove that the mechanisms obtained by HSATS, HGLS, and SPA have the property of "price non-negativity", the prices obtained via the three algorithms are all non-negative in these numerical examples. The property "incentive compatibility" is not tested because it is impossible to enumerate all combinations of passengers' reported requirements, but the mechanism obtained

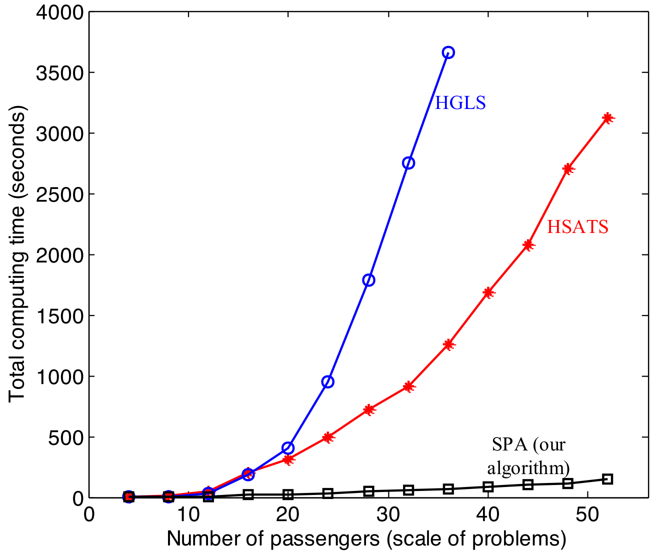


Fig. 4. Computing time of HSATS, HGLS, and SPA for different numerical examples.

Table 4
Comparison results of the properties of “individual rationality” and “price non-negativity”.

Numerical examples	Percentage of “individual rational” prices				Percentage of non-negative prices			
	EA	HSATS	HGLS	SPA	EA	HSATS	HGLS	SPA
N_4	Proved (100)	100	100	Proved (100)	Proved (100)	100	100	100
N_8	Proved (100)	100	100	Proved (100)	Proved (100)	100	100	100
N_12	Proved	100	100	Proved (100)	Proved	100	100	100
N_16	Proved	100	100	Proved (100)	Proved	100	100	100
N_20	Proved	100	100	Proved (100)	Proved	100	100	100
N_24	Proved	**91.7**	100	Proved (100)	Proved	100	100	100
N_28	Proved	100	100	Proved (100)	Proved	100	100	100
N_32	Proved	100	**96.9**	Proved (100)	Proved	100	100	100
N_36	Proved	100	100	Proved (100)	Proved	100	100	100
N_40	Proved	100		Proved (100)	Proved		100	100
N_44	Proved	100		Proved (100)	Proved		100	100
N_48	Proved	**97.9**		Proved (100)	Proved		100	100
N_52	Proved	100		Proved (100)	Proved		100	100

Note: the table only presents the data when the computer memory is sufficient and the computing time is less than one hour (3600 s).

by SPA has been proved to be incentive compatible (Proposition 2), while the mechanism obtained by other heuristics (e.g. HSATS and HGLS) is not incentive compatible based on the discussion in Section 2.3.

Table 5 shows that the profits (total price collected minus total transportation cost) are positive for all numerical examples in the mechanisms obtained by EA, HSATS, HGLS, and SPA. The mechanisms obtained by EA, HSATS, HGLS, and SPA are all profitable for the service provider in all of the numerical examples.

4.5. Sensitivity analysis

Sensitivity analysis focuses on two aspects: 1) change of passengers' value functions and 2) change of the strictness of passengers' requirements on inconvenience factors. The first aspect aims at testing the effectiveness of the mechanism under different conditions, in which passengers have stricter attitudes towards the price. The second aspect is to study the changing process of the matching and routing plan and the price when a passenger in one location places stricter requirement on the inconvenience factors.

1) Change of the value function

Passengers' attitudes towards the price are reflected by the value function. We use a different hypothetical value function Formula (14) instead of Formula (13) to represent passengers' stricter attitudes towards the price. Formula (14) assumes that passengers' lowest maximum willing-to-pay price is zero if they are transported to the transit hub directly, i.e. $V_i \geq 0$,

Table 5
Profit made by the ridesharing service provider.

Numerical examples	Profit made by the service provider (total price minus total transportation cost)			
	EA	HSATS	HGLS	SPA
N_4	19.9	19.9	19.9	19.2
N_8	35.2	35.2	35.2	35.2
N_12		59.4	59.4	59.4
N_16		79.8	79.8	80.8
N_20		94.4	94.4	94.2
N_24		113.5	112.7	106.9
N_28		136.7	136.4	134.1
N_32		145.0	147.8	144.4
N_36		169.1	158.3	160.3
N_40		194.7		191.2
N_44		207.4		186.0
N_48		239.1		227.0
N_52		236.1		234.7

Note: the table only presents the results when the computer memory is sufficient and the computing time is less than one hour (3600 s).

Table 6
Objective function values of IP0 obtained by EA, HSATS, HGLS, and SPA (value function: Formula (14)).

Numerical examples	EA	HSATS	HGLS	SPA		
				Objective function values	Difference from HSATS (%)	Difference from HGLS (%)
N2_4	26.17	26.17	26.17	26.17	0.00	0.00
N2_8	48.00	48.00	48.00	48.00	0.00	0.00
N2_12		71.21	71.21	71.21	0.00	0.00
N2_16		98.56	98.56	98.56	0.00	0.00
N2_20		130.83	130.83	130.75	0.06	0.06
N2_24		169.09	169.09	168.44	0.39	0.39
N2_28		181.21	181.21	176.32	2.77	2.77
N2_32		199.44	199.38	197.86	0.80	0.77
N2_36		248.52	248.52	245.43	1.26	1.26
N2_40		253.24	252.88	246.61	2.69	2.54
N2_44		260.08	260.08	256.26	1.49	1.49
N2_48		314.25	314.54	311.52	0.88	0.97
N2_52		336.39	335.58	333.83	0.77	0.52

Table 7
Computing time of HSATS, HGLS, and SPA (value function: Formula (14)).

Numerical examples	HSATS			HGLS			SPA		
	TO (s)	TP (s)	TT (s)	TO (s)	TP (s)	TT (s)	TO (s)	TP (s)	TT (s)
N2_4	0.10	0.42	0.52	0.09	0.33	0.42	0.13	0.01	0.14
N2_8	1.14	7.98	9.12	0.72	5.65	6.37	1.44	0.31	1.75
N2_12	3.65	40.59	44.24	2.43	29.55	31.98	4.79	1.26	6.05
N2_16	10.34	159.98	170.32	11.47	172.75	184.22	13.96	3.50	17.46
N2_20	14.00	271.03	285.03	19.46	360.47	379.93	17.89	6.96	24.85
N2_24	17.94	441.51	459.45	37.44	901.17	938.61	22.84	9.64	32.48
N2_28	23.92	628.69	652.61	66.91	1729.90	1796.81	29.70	14.24	43.94
N2_32	27.93	907.01	934.94	85.38	2612.03	2697.41	34.11	16.96	51.07
N2_36	35.77	1266.50	1302.27	94.75	3635.89	3730.64	42.20	23.88	66.08
N2_40	40.68	1411.74	1452.42	109.55	>3600	>3600	51.65	25.36	77.01
N2_44	46.43	2090.88	2137.31	131.43	>3600	>3600	55.80	36.93	92.73
N2_48	53.84	2274.47	2328.31	157.79	>3600	>3600	71.03	47.81	118.84
N2_52	58.74	2620.70	2679.44	182.50	>3600	>3600	89.79	53.69	143.48

if $IVT_i = t_{i0}$.

$$V_i = V_{\max}^i - \left(\frac{\alpha_i^{NR} NR_i}{(Q - np_i)} + \frac{\alpha_i^{IVT}(IVT_i - t_{i0})}{t_{i0}} + \frac{\alpha_i^{WT} WT_i}{MD} \right) \frac{V_{\max}^i}{2} \tag{14}$$

We will test the mechanism using the same algorithms. The experiment results are listed in Tables 6–9. The numerical examples are denoted as “N2_x”, where x represents the number of requests sent by passengers. Yet again, ANTIGONE is not presented in Tables 6–9 due to its unreasonably long computing time.

Table 8
Comparison results of the property of "individual rationality" and "price non-negativity" (value function: Formula (14)).

Numerical examples	Percentage of "individual rational" prices				Percentage of non-negative prices			
	EA	HSATS	HGLS	SPA	EA	HSATS	HGLS	SPA
N2_4	Proved (100)	100.0	100.0	Proved (100)	Proved (100)	100.0	100.0	100.0
N2_8	Proved (100)	100.0	100.0	Proved (100)	Proved (100)	100.0	100.0	100.0
N2_12	Proved	100.0	100.0	Proved (100)	Proved	100.0	100.0	100.0
N2_16	Proved	100.0	100.0	Proved (100)	Proved	100.0	100.0	100.0
N2_20	Proved	100.0	100.0	Proved (100)	Proved	100.0	100.0	100.0
N2_24	Proved	100.0	100.0	Proved (100)	Proved	100.0	100.0	100.0
N2_28	Proved	100.0	100.0	Proved (100)	Proved	100.0	100.0	100.0
N2_32	Proved	100.0	**84.4**	Proved (100)	Proved	100.0	100.0	100.0
N2_36	Proved	100.0	100.0	Proved (100)	Proved	100.0	100.0	100.0
N2_40	Proved	100.0		Proved (100)	Proved	100.0		100.0
N2_44	Proved	100.0		Proved (100)	Proved	100.0		100.0
N2_48	Proved	**95.8**		Proved (100)	Proved	100.0		100.0
N2_52	Proved	100.0		Proved (100)	Proved	100.0		100.0

Note: the table only presents the data when the computer memory is sufficient and the computing time is less than one hour (3600 s).

Table 9
Profit made by the ridesharing service provider (value function: Formula (14)).

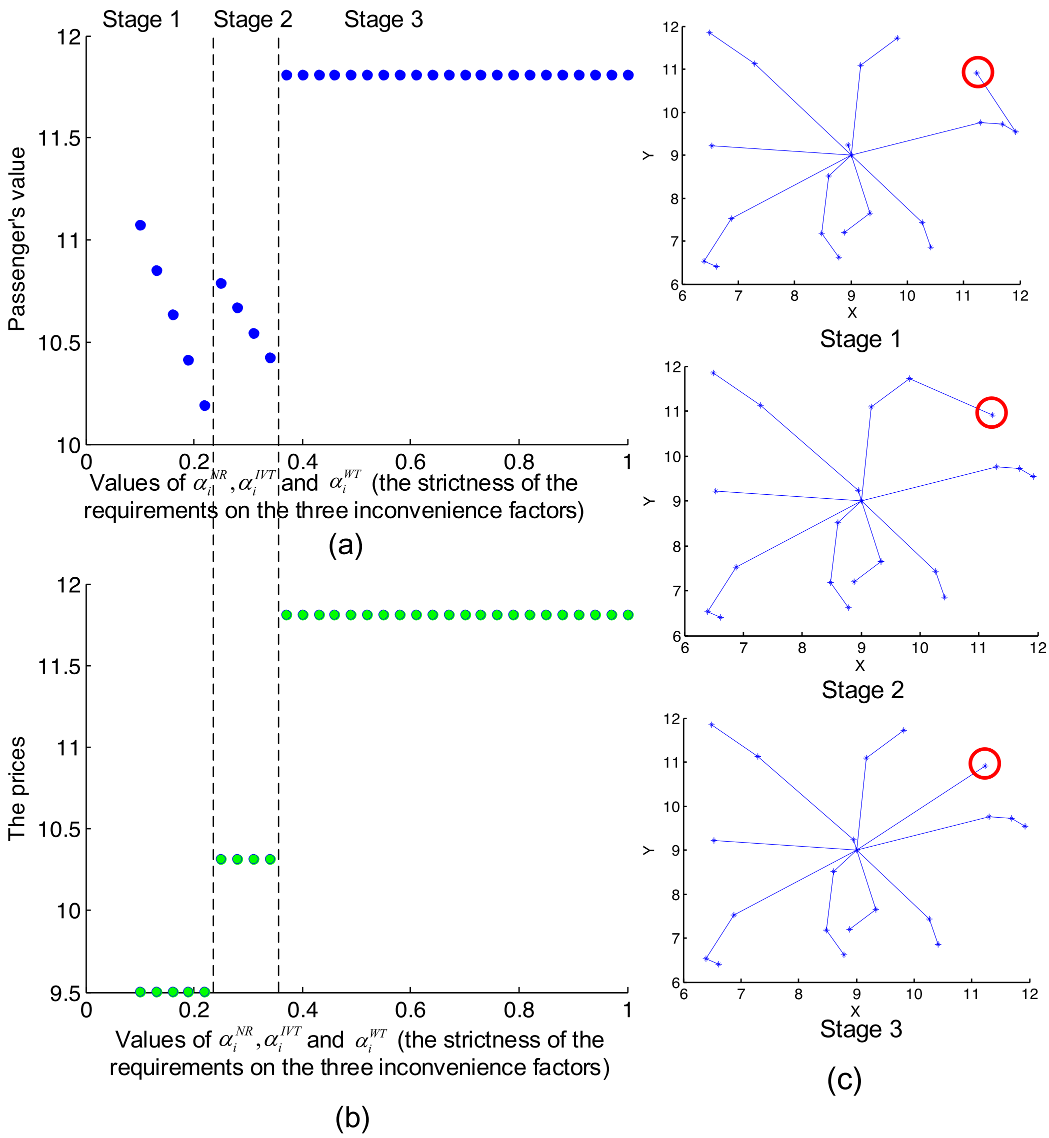
Numerical examples	Profit made by the service provider (total price minus total transportation cost)			
	EA	HSATS	HGLS	SPA
N2_4	22.30	22.30	22.30	22.30
N2_8	41.20	41.20	41.20	41.20
N2_12		59.50	59.50	59.40
N2_16		72.60	72.60	67.10
N2_20		96.00	96.00	92.80
N2_24		117.90	118.10	113.20
N2_28		141.40	141.40	115.80
N2_32		153.60	155.00	141.90
N2_36		173.90	172.70	143.10
N2_40		192.00		149.20
N2_44		200.80		186.50
N2_48		247.90		209.50
N2_52		242.40		147.60

Note: the table only presents the data when the computer memory is sufficient and the computing time is less than one hour (3600 s).

Tables 6 and 7 present the comparison results in terms of the objective function value and computing time. SPA can still obtain satisfactory vehicle-passenger matching and routing plans within a reasonable time when passengers' value functions change. The results show that when the scale of the problem is small, HSATS, HGLS, and SPA are able to obtain the exact optimal solution. With the scale of the problem increasing, the solution qualities of HSATS and HGLS are slightly higher than those of SPA, but the differences between SPA and HSATS and between SPA and HGLS are still negligible. The largest difference between SPA and HSATS/HGLS in terms of the objective function value is only 2.77%. The total computing time of SPA is less than 3 min, significantly less than those of HSATS and HGLS. In Table 8, HSATS and HGLS may generate mechanisms that are not individual rational (numerical examples N2_48 and N2_32 with bold numbers). All of the prices obtained by HSATS, HGLS, and SPA are still non-negative even though passengers' attitudes towards prices becomes stricter. Table 9 shows the profits of all numerical examples based on the mechanism obtained by HSATS, HGLS, and SPA. All profits are positive, indicating that even though the passengers have stricter attitudes towards the price, the mechanism is still profitable for the service provider. The experimental results demonstrate the robustness of SPA even if passengers' attitude towards the prices changes.

1) Change of passengers' tolerance for inconvenience factors

This sensitivity analysis studies the impact of changing passenger's requirements on prices and the matching and routing plans. We state that the changing process of the mechanism $M(X^{IP_0*}, \mathbf{p})$ is *reasonable* if the passenger receives no worse service and the price does not decrease when the requirement becomes stricter. Fig. 5 shows an example of a *reasonable* changing process of one passenger's (this passenger is highlighted by the red circle in the figure) mechanism $M(X^{IP_0*}, p_i)$. There are three stages of the changing process in Fig. 5. The price and the matching and routing plan do not change within each stage. As the passenger's requirements continue to grow stricter, the stage will transition to the next stage, and the passenger will receive higher-quality service and the price increases. The reasonable changing process is important because



(a) Changing process of the passenger's value
(b) Changing process of the price
(c) Changing process of the routing plan

Fig. 5. An example of reasonable changing process of one passenger's mechanism.

it avoids the following counter-situation: a passenger places a stricter requirement on an inconvenience attribute, but has to tolerate an increased degree of the corresponding inconvenience attribute and pays less money.

In the sensitivity analysis, the values of $\alpha_i{}^{NR}$, $\alpha_i{}^{IVT}$, and $\alpha_i{}^{WT}$ are all increased from 0.1 to 1 by 0.1 each time for each passenger. We solve the mechanism $M(X^{IP_0*}, \mathbf{p})$ each time $\alpha_i{}^{NR}$, $\alpha_i{}^{IVT}$, and $\alpha_i{}^{WT}$ increase. We record the number of passenger requests whose mechanism changing processes are *reasonable* and calculate the percentage of this number in the total number of passenger requests for each numerical example.

Table 10 shows the percentages of the number of passenger requests, whose changing processes of the mechanism are *reasonable*, in the total number of passenger requests. When the scale of the problems is small, all algorithms can ensure 100% *reasonable* changing processes. However, as the scale of problems increases, these percentages of regular heuristic algorithms, including HSATS and HGLS, decrease sharply (see Fig. 6). Thus, when the scale of the problem is large, even though passengers' requirements become stricter, the routing plan is likely to become less convenient for such passengers

Table 10
Percentages of reasonable changing processes.

Numerical examples	Percentages of reasonable changing processes			
	EA	HSATS	HGLS	SPA
N_4	100	100.0	100.0	100.0
N_8	100	100.0	100.0	100.0
N_12		100.0	100.0	100.0
N_16		100.0	100.0	100.0
N_20		100.0	90.0	100.0
N_24		16.7	25.0	100.0
N_28		14.3	14.3	100.0
N_32		18.8	18.8	100.0
N_36		0.0	0.0	100.0
N_40		12.5		100.0
N_44		6.8		100.0
N_48		6.3		100.0
N_52		0.0		100.0
N2_4	100	100.0	100.0	100.0
N2_8	100	100.0	100.0	100.0
N2_12		100.0	100.0	100.0
N2_16		100.0	100.0	100.0
N2_20		95.0	100.0	100.0
N2_24		66.7	54.2	100.0
N2_28		96.4	89.3	100.0
N2_32		15.6	25.0	100.0
N2_36		19.4	16.7	100.0
N2_40		30.0		100.0
N2_44		27.3		100.0
N2_48		0.0		100.0
N2_52		3.8		100.0

Note: the table only presents the data when the computer memory is sufficient and the computing time is less than one hour (3600 s).

and the price will decrease, which counteracts the mechanism design objective. For example, if a passenger places stricter requirement on the extra in-vehicle travel time, the system is likely to let her stay in the vehicle for a longer time and the price is likely to decrease by using HSATS or HGLS. In contrast, from the testing result, it seems that SPA can always ensure a *reasonable* changing process of the mechanism for all passengers (100%) in all of the numerical examples.

5. Conclusions

This paper proposes a novel heuristic algorithm, the Solution Pooling Approach (SPA), to obtain the mechanism proposed in our Part I paper. The SPA is able to ensure two important properties, “individual rationality” and “incentive compatibility”. The experimental results of the numerical example show that the SPA can significantly decrease the computational complexity with a minimal sacrifice of solution quality, compared with commercial solvers (e.g. ANTIGONE) and traditional heuristic methods, such as the Hybrid Simulated Annealing–Tabu Search Algorithm and the Hybrid Genetic Algorithm. From the sensitivity analysis, we can conclude that the SPA is robust enough to efficiently obtain the mechanism without sacrificing too much accuracy and to maintain some other desirable properties, including price non-negativity and service provider profitability based on the numerical examples. The sensitivity analysis also suggests that passengers can receive a higher-quality service by placing stricter requirements on corresponding inconvenience factors based on their mobility preferences, and correspondingly, they are charged a higher price when participating in ridesharing. The SPA can be adapted to solve generalized mechanism design problems. We analyze the specific circumstances under which the SPA can sustain the game-theoretic properties, including “individual rationality” and “incentive compatibility”, and identify its limitations in solving generalized mechanism design problems. Our future work will apply the solution pooling approach to solve other mechanism design problems and test its effectiveness.

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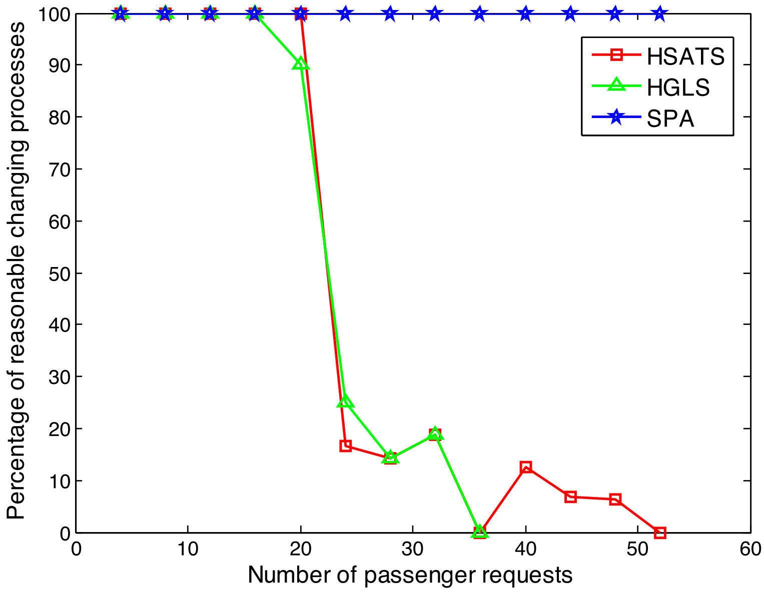
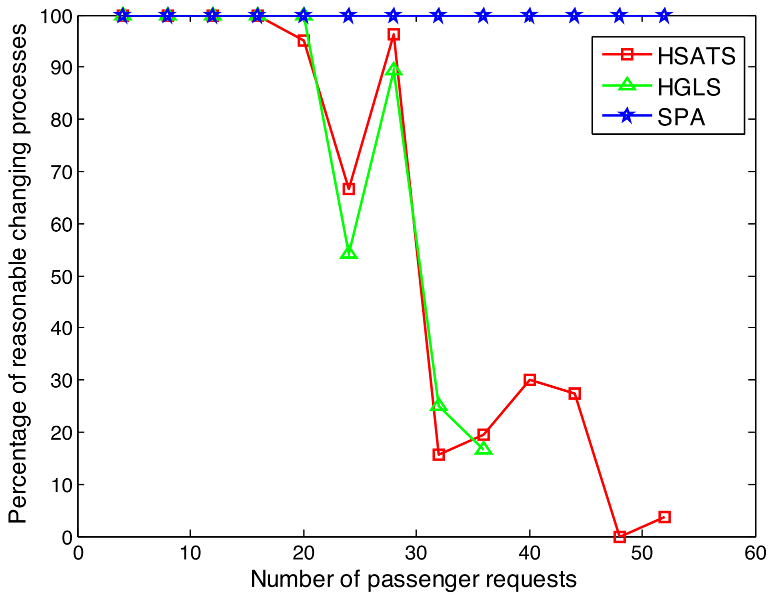
Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.trb.2018.12.014.



(a) Value function: Formula (13)



(b) Value function: Formula (14)

Fig. 6. Percentages of number of *reasonable* changing processes for different numerical examples.

Appendix A

Table A-1.
Formulas in the models IP_0 and IP_g

$$\min \sum_{i \in P} C_i^{ICN}(NR_i, IVT_i, WT_i, \alpha_i^{NR}, \alpha_i^{IVT}, \alpha_i^{WT}) + TC(X) \tag{A.1}$$

$$\max Z_0(X) = \sum_{i \in P} V_i(X) - TC(X) \tag{A.2}$$

Table A-1
Notations.

Sets	
P	Set of passenger requests, $P=\{1, 2, \ldots, n\}$
V	Set of vehicles, $V=\{1, 2, \ldots, m\}$
H	Set of the transit hub, $H=\{0\}$
Variables	
$x_{ijk} = \begin{cases} 1, \text{ if vehicle } k \text{ travels to location } j \text{ after picking up passenger(s) in location } i \text{ immediately} \\ 0, \text{ otherwise} \end{cases}$	$i \in P,\ j \in P \cup H,\ k \in V$
$y_{ik} = \begin{cases} 1, \text{ if vehicle } k \text{ is dispatched to pick up passenger(s) in location } i \\ 0, \text{otherwise} \end{cases}$	$i \in P,\ k \in V$
$X=\{x_{ijk}, y_{ik} \mid i \in P, j \in P \cup H, k \in V\}$ can represent a vehicle-passenger matching and vehicle routing plan.	
$w_{ik} = \begin{cases} 1, \text{ if passenger(s)in location} i \text{ is the first to be picked up by vehicle } k \\ 0, \text{otherwise} \end{cases}$	$i \in P,\ k \in V$
NR_i	Number of co-riders with passenger(s) i.
IVT_i	Passenger(s) i's in-vehicle travel time.
WT_i	Passenger(s) i's extra waiting time at the transit hub
C_i^{ICN}	Passenger(s) i's inconvenience cost caused by ridesharing.
$V_i(X)$	Passenger(s) i's value gained from the ridesharing service given a routing plan X. V_i can also be treated as the maximum price that this passenger is willing to pay.
Parameters	
np_i	Number of passengers in request i. For denotation convenience, we let "passenger(s) i" represent the passenger(s) in request i.
DL_i	Passenger(s) i's preferred deadline before which he/she/they must arrive at the transit hub.
t_{ij}	The travel time from node i to node j, i and $j \in P \cup H$. The pickup time is included in t_{ij}.
c_{ij}	The transportation cost from node i to node j, i and $j \in P \cup H$.
Q	The seat capacity of a vehicle, excluding the driver.
α_i^{NR}, α_i^{IVT}, and α_i^{WT}	Passenger(s) i's personalized requirements on the number of shared riders, in-vehicle travel time, and extra waiting time at the transit hub, respectively. The three parameters are obtained from passengers' reported information. The personalized requirements, represented by α_i^{NR}, α_i^{IVT}, and α_i^{W} can be any form, as long as the three parameters can convey passengers' different tolerances for the inconvenience attributes.
$V^i_{\max}$	Passenger(s) i's maximum willing-to-pay price when she is transported from the origin to the transit hub directly without any shared riders.

where

$$V_i(X) = V^i_{\max} - C^{ICN}\left(NR_i(X), IVT_i(X), WT_i(X), \alpha_i^{NR}, \alpha_i^{IVT}, \alpha_i^{WT}\right)$$

$$TC(X) = \sum_{k\in V}\sum_{i\in P}\sum_{j\in P\cup H\setminus i} x_{ijk}c_{ij}$$

Subject to

$$\sum_{k\in V} y_{ik} = 1, \text{ for all } i \in P \tag{A.3}$$

$$\sum_{i\in P} y_{ik}np_i \le Q, \text{for all } k \in V \tag{A.4}$$

$$w_{ik} + \sum_{i\in P\setminus j} x_{ijk} = y_{jk}, \text{ for all } k \in V, j \in P \tag{A.5}$$

$$\sum_{j\in P\cup H\setminus i} x_{ijk} = y_{ik}, \text{ for all } k \in V, i \in P \tag{A.6}$$

$$\sum_{i\in P} w_{ik} \le 1, \text{ for all } k \in V \tag{A.7}$$

$$IVT_i = \sum_{k\in V}\sum_{j\in H\cup P\setminus i} x_{ijk}\left(IVT_j + t_{ij}\right), \text{ for all } i \in P \tag{A.8}$$

$$IVT_i \ge 0, \text{ for all } i \in P \tag{A.9}$$

$$WT_i = DL_i - \min_{j \in P} \left\{ M\left(1 - \sum_{k \in V} y_{jk} y_{ik}\right) + DL_j \right\}, \text{for all } i \in P \tag{A.10}$$

$$NR_i = \sum_{j \in P \backslash i} \sum_{k \in V} y_{ik} y_{jk} np_j, \text{ for all } i \in P \tag{A.11}$$

$$x_{ijk}, y_{ik}, w_{ik} \in \{0, 1\}, \text{ for all } i, j \in P \cup H, k \in V \tag{A.12}$$

$$NR_g = 0 \tag{A.13}$$

Formula (A.1) is the objective function of minimizing passengers' inconvenience cost and the service provider's transportation cost, which is equivalent to Formula (A.2) that maximizes passengers' accumulative values minus the agency's transportation cost. A passenger's value is defined as the maximum willing-to-pay price, non-inconvenience value minus the inconvenience cost. A passenger's inconvenience cost is a function of the number of co-riders, in-vehicle travel time, and extra waiting time at the transit hub. Formula (A.3) ensures that all passengers will be picked up by one vehicle and only be served once. Formula (A.4) represents that the maximum capacity of each vehicle should not be exceeded. Formulas (A.5) and (A.6) ensure the balanced flow from and to each passenger location. Formula (A.7) ensures that each vehicle can only be dispatched once at most. Formula (A.8) gets all passengers' in-vehicle travel times. Formula (A.9) is to ensure the non-negativity of all passengers' in-vehicle travel times. Formulas (A.10) and (A.11) get all passengers' extra waiting times at the transit hub and the number of shared riders, respectively. Formula (A.12) signifies that x_{ijk}, y_{ijk}, and w_{ik} are binary variables. Formula (A.13) ensures passenger(s) g is transported directly to the transit hub without shared co-riders.

Appendix B

This appendix presents the performance of seven commercial solvers, ANTIGONE (Algorithms for coNTinuous/Integer Global Optimization), ALPHAECP (α-Extended Cutting Plane), BARON (Branch-And-Reduce Optimization Navigator), COUENNE (Convex Over and Under ENvelopes for Nonlinear Estimation), LINDOGLOBAL, SBB (Simple Branch-and-Bound) and SCIP (Solving Constraint Integer Programs), in terms of the objective function values and the computing time in solving the non-convex mixed integer non-linear programming model IP_0. Among these seven solvers, ANTIGONE, COUENNE, LINDOGLOBAL, and SCIP can guarantee the global optimal solutions for non-convex MINLP models if the solvers are terminated normally, while ALPHAECP, BARON, and SBB cannot ensure the global optimality (Bussieck and Vigerske, 2010). The results are shown in Tables B-1 and B-2. The two tables do not show the result when the solvers are unable to return a solution.

Table B-1
Objective function values of model IP_0 obtained by seven solvers.

Numerical examples	Objective function values (dollars)						
	ANTIGONE	ALPHAECP	BARON	COUENNE	LINDOGLOBAL	SBB	SCIP
N_4	24.56	24.56	24.56	24.56	24.56	24.56	24.56
N_8	58.00	58.00	58.00	58.00	58.00	55.30	58.00
N_12	81.52	81.52	81.52	81.11	81.52	78.44	81.52
N_16	119.98	115.36	117.57				116.79
N_20	139.89	134.13	131.11				136.51
N_24	152.58	139.59	140.73				133.72
N_28	159.28						147.18

Table B-2
Computing times of seven solvers in solving model IP_0.

Numerical examples	Computing time (seconds)						
	ANTIGONE	ALPHAECP	BARON	COUENNE	LINDOGLOBAL	SBB	SCIP
N_4	0.09	8.07	0.15	0.87	0.28	0.17	0.20
N_8	1.95	423.69	8.55	139.39	21.46	12.36	5.37
N_12	2278.09	3600.00	3600.00	3600.00	3600.00	441.02	3600.00
N_16	3600.00	3600.00	3600.00				3600.00
N_20	3600.00	3600.00	3600.00				3600.00
N_24	3600.00	3600.00	3600.00				3600.00
N_28	3600.00						3600.00

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