



Mechanism design for first-mile ridesharing based on personalized requirements part I: Theoretical analysis in generalized scenarios

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ABSTRACT

Ridesharing is an effective transportation mode to provide first-mile accessibility to public transit and a low-cost, environment-friendly, and sustainable mobility service. This paper designs a mechanism for the first-mile ridesharing service. The mechanism accounts for passengers' personalized requirements on different inconvenience attributes (e.g. the number of co-riders, extra in-vehicle travel time, and extra waiting time at the transit hub) of the service in determining the optimal vehicle-passenger matching and vehicle routing plan and customized pricing scheme. The proposed mechanism is proved to be individual rational, incentive compatible, and price non-negative. The three properties respectively indicate that passengers are willing to participate in the service, that honestly reporting personalized requirements is the optimal strategy, and that the service provider is guaranteed to receive revenue from the participants. A case study is proposed to interpret the mechanism and to demonstrate the generality of the personalized-requirement-based mechanism that can be adapted into different scenarios.

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1. Introduction

Americans take 11 billion trips annually on public transportation, a 40% increase since 1995 (American Public Transportation Association, 2016). The American public transportation industry faces an ongoing challenge of transit hub accessibility – how travelers get to nearby transit hubs. This challenge is also known as the “first-mile” bottleneck. Several studies have found that travelers' choices of public transportation are significantly affected by the accessibility to transit hubs (Krygsman et al., 2004; Rietveld, 2000). In the United States, many transit riders either drive their own vehicles or take taxis or other emerging mobility services (e.g. Uber and Lyft) to nearby transit hubs. However, uncoordinated traveling does not fully utilize the empty seats in a car, which in turn increases traffic congestion, emissions, and parking demands.

Ridesharing is a potential solution to address first- or last-mile transit accessibility, and to provide a low-cost, environment-friendly and sustainable mobility service (Furuhata et al., 2013; Cici et al., 2014). There are various types of ridesharing services. Furuahata et al. (2013) classified ridesharing into three categories, carpooling/vanpooling, long-distance ride-match, and dynamic real-time ridesharing based on target markets. Furuahata et al. (2013) indicated that carpooling usually targets commuters and that users can schedule the service. Long-distance ride-match provides intercity or interstate

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trips. This service usually requires passengers to schedule the service in advance. Real-time dynamic ridesharing provides an automated process of ride-matching between drivers and passengers on very short notice or even en-route. Thus, based on the user type, ridesharing can be categorized as scheduled and on-demand services. For scheduled service, passengers send requests early enough (e.g. at least 30 minutes) before they need the service. The system can pre-optimize the matching and routing plan and pre-determine the prices before the service is approaching. For on-demand service, passengers send spontaneous requests when they need the service. The system needs to optimize the matching and routing plan and determine the prices in real time, so that vehicles can be dispatched to serve passengers within a very short time. Ridesharing can also be categorized as targeted and untargeted services. Targeted ridesharing provides the service for specific type of passengers (e.g. commuters, transit riders, etc.). Passengers using a targeted ridesharing service usually have common destinations (e.g. companies, transit hubs, etc.). Untargeted ridesharing provides the service for any passenger who sends a request. Passengers using an untargeted ridesharing service usually have different destinations. In this paper, we focus on the scheduled first-mile ridesharing to the transit hub, accounting for its characteristics.

The prior literature has recognized the trend of integrating first-mile ridesharing with public transportation. For example, [Shaheen and Chan \(2016\)](#) discussed how mobile technology and public policy continue to evolve to integrate shared mobility with public transit. [Masoud et al. \(2017a\)](#) developed a mobile application with an innovative ride-matching algorithm as a decision support tool that suggests transit-rideshare connections. [Stiglic et al. \(2018\)](#)'s study showed that the integration of a ridesharing system and a public transit system can significantly enhance mobility and increase the use of public transport. [Ma \(2017\)](#) proposed a dynamic bi-modal vehicle dispatching and routing algorithm to address the real-time operating policy of ridesharing (feeder) services in coordination with the presence of existing public transportation networks. In addition, there is potentially a high demand for the first-mile ridesharing service in transit-intensive metropolitan areas. For example, based on the NYC taxicab data ([New York City Taxi, & Limousine Commission, 2018](#)), there were 3,122,731 taxi trips to the Pennsylvania Station in New York City in 2017. An average of 8555 taxis traveled to this station every day. Among 3,122,731 taxi trips, 2,189,467 trips (70.1%) had only one passenger per trip. Among these one-passenger trips, approximately 1,509,580 (68.95%) taxi trips are within the same pickup zone and their pickup times are within 10 minutes. These trips might potentially be combined under certain incentive mechanisms for ridesharing. Also, ridesharing service providers (e.g. Uber and Lyft) have already added public transportation to their apps, allowing for seamless transfers from their ridesharing to the public transit services for convenient multi-modal journeys ([Shelton, 2016](#); [Smartrail World, 2016, 2018](#)) in New York, Boston, Los Angeles, and other metropolitan cities around the world. This market of emerging multi-modal first-mile ridesharing service inspires us to design a mechanism to incentivize more passengers to participate in ridesharing.

Mechanism design theory is a field in economics and game theory that designs economic mechanisms or incentives toward desired objectives ([Hurwicz and Reiter, 2006](#)). The proposed mechanism includes an optimal vehicle-passenger matching and vehicle routing plan as well as a customized pricing strategy. The matching and routing plan determines each passenger's personalized first-mile ridesharing service. The customized pricing strategy provides passengers with economic incentives to participate in ridesharing by offsetting the inconvenience caused by ridesharing. Our designed mechanism allows passengers to detail their personalized requirements on the following so-called "inconvenience factors": (1) extra in-vehicle travel time (for example, detour to pick up other passengers); (2) the number of co-riders sharing the vehicle; and (3) extra waiting time at the transit hub due to potential early arrival. Previous studies ([Golledge et al., 1994](#); [Ben-Akiva and Lerman, 1985](#); [Arentze, 2013](#)) recognized that travelers' choice of transportation mode is not only influenced by price, but also by these "inconvenience" attributes. The methodology can be adapted to account for additional factors in future research. The proposed mechanism can promote passengers' participation by ensuring an important property, "individual rationality", which indicates that passengers' maximum willing-to-pay prices will never be exceeded by the actual paid prices. In addition, rational passengers may misreport their personalized requirements in order to maximize their utilities if the mechanism cannot prevent this. Thus, the designed mechanism needs to ensure another important property, namely "incentive compatibility", representing that truthfully reporting the requirement is each passenger's optimal strategy, which maximizes the utility. This property can prevent passengers from misreporting their personalized requirements. Moreover, the price should be non-negative, so that the service provider can gain revenue from passengers. Finally, a case study is proposed to interpret the mechanism and to demonstrate the effectiveness of the proposed mechanism.

This paper is structured as follows. We start with reviewing the literature on mechanism design and shared mobility to identify knowledge gaps and research needs in [Section 2](#). Then, we introduce our designed mechanism in [Section 3](#). In [Section 4](#), a case study is proposed to interpret the potential application of the proposed mechanism. Concluding remarks are made in [Section 5](#) and future work is introduced in [Section 6](#).

2. Literature review

2.1. Existing work

Much prior work has focused on vehicle-passenger matching and vehicle routing optimization issues in the field of ridesharing research. Different algorithms, such as Lagrangean column generation, the genetic heuristic algorithm, particle swarm optimization, and the rolling horizon planning approach, were developed to solve both static and dynamic matching

and/or routing problems for ridesharing services (Baldacci et al., 2004; Calvo et al., 2004; Agatz et al., 2011; Ghoseiri et al., 2011; Armant and Brown, 2014; Ma et al., 2013; Wang et al., 2017; Huang et al., 2015; Ozkan and Ward, 2016; Jung et al., 2016; Ma, 2017; Huang et al., 2018; Masoud and Jayakrishnan, 2017a, 2017b; Agussurja et al., 2018; Chen et al., 2018; Hou et al., 2018; Wang et al., 2018b etc.). However, these studies have not addressed customized pricing for each individual passenger nor measured the impact of the pricing mechanism on vehicle-passenger matching as well as the vehicle routing plan.

Proper pricing is a key incentive to promote passengers' collaboration to share the ride. There are various pricing strategies in the literature that can be classified into four categories, supply-demand-balance pricing strategy, fair cost allocation strategy, pricing optimization strategy, and auction-based pricing mechanisms.

The first type of strategy is supply-demand-balance strategy, which is widely used in taxi services (Yang et al., 2002; Zhang and Ukkusuri, 2016; Qian and Ukkusuri, 2017). Researchers applied and modified the pricing strategy to adapt into ridesharing services (Witt et al., 2015; Banerjee et al., 2015; Fang et al., 2017; Liu and Li, 2017; Zha et al., 2017; Yang et al., 2018). The mechanism of this pricing strategy can be described as follows. When customers' demand exceeds the supply, the price is increased to re-balance the demand and supply, and vice versa. Ridesharing companies, like Uber and Lyft, use this pricing framework to incentivize drivers to move to undersupplied locations (Hall et al., 2015). Since this pricing scheme is determined by the relationship between supply and demand, passengers' prices are not directly related to the matching and routing plan and some inconvenience factors (e.g. number of shared riders and extra in-vehicle travel time).

The second type of strategy fairly allocates costs among passengers. This pricing scheme is inspired by the work of Frisk et al. (2010) on cost allocation in collaborative forest transportation. Subsequently, researchers proposed fair pricing schemes (Bistaffa et al., 2015; Gopalakrishnan et al., 2016; Li et al., 2016; Wang et al., 2018a; Peng et al., 2018) based on different travel attributes, such as travel distance, detours, and waiting time. The fair cost-sharing mechanism, however, does not have the function that allows passengers to report their preferences over these attributes (e.g. via smartphone applications).

The third strategy optimizes the pricing along with the matching and/or routing to achieve certain objectives, such as maximizing the total profit, minimizing passengers' total travel cost, and maximizing the total saved travel mileage. For example, Cheng et al. (2012) optimized the vehicle-passenger matching and passengers' prices with the objective of minimizing the total travel cost, including the monetary payments and time penalties. Biswas et al. (2017) attempted to maximize the total profit made by the service provider by determining the optimal prices and matching and routing plans. Santos and Xavier (2015) solved a dynamic ridesharing problem and designed an optimal incentive for passengers' participation so that the number of served requests is maximized and the sum of all served requests' costs is minimized. Qian et al. (2017) investigated the optimal assignment of a set of passengers for the sake of maximizing total saved travel miles, analyzed different behaviors of passengers and drivers in participating taxi group rides, and explored the best incentives for taxi ridesharing in order to maximize efficiency under the optimal assignment. Chen and Wang (2018) optimized the price, number of operation vehicles, and vehicle capacity to maximize the social welfare for last-mile ridesharing, without considering the matching and routing optimization. Nevertheless, very few prior studies fully investigated whether or not the designed incentive can offset the inconvenience caused by ridesharing considering passengers' personalized requirements in order to promote ridesharing participation.

The fourth type of strategy is auction-based pricing mechanisms that aim to maximize society's overall welfare by requiring participants to truthfully report their valuations of the service. The Vickrey–Clarke–Groves (VCG) mechanism is a widely used truth-inducing mechanism, which aims to maximize the cumulative value of collaboration (Vickrey, 1961; Clarke, 1971; Groves, 1973). However, the VCG mechanism has inherent limitations in the application of ridesharing. It is not budget-balanced and requires external subsidies to sustain the service (Parkes et al., 2001). Besides, calculating VCG payments is computationally challenging (Kamar and Horvitz, 2009). Thus, various auction-based mechanisms (Kamar and Horvitz, 2009; Cheng et al., 2014; Zhao et al., 2014; Zhao et al., 2015; Asghari et al., 2016; Asghari and Shahabi, 2017; Shen et al., 2016; Nguyen, 2013; Zhang et al., 2016; Kleiner et al., 2011; Masoud et al., 2017b; Masoud and Lloret-Batlle, 2016; Ma et al., 2018; Hsieh et al., 2018; Zhang et al., 2018) are designed to modify or replace the VCG mechanisms in different scenarios. Although these mechanisms incentivize passengers to truthfully report their valuations of the ridesharing service, none of them allow passengers to input their personalized requirements on inconvenience factors caused by ridesharing. Thus the incentive mechanisms are not based on passengers' personalized requirements.

2.2. Knowledge gaps and intended contributions

To our knowledge, very little prior research has addressed the incentive mechanism design for first-mile ridesharing with respect to public transit accessibility. First-mile ridesharing has four characteristics: (1) all passengers have the same destination (i.e., the transit hub); (2) passengers may have a strict deadline for arriving at the transit hub; (3) passengers can schedule the first-mile ridesharing service in advance if they know their transit schedules (particularly for commuters); and (4) in addition to the number of shared riders and extra in-vehicle travel time, first-mile ridesharing imposes upon passengers another potential inconvenience factor, being the extra waiting time at the transit hub due to early arrival if passengers served by the same vehicle have different arrival deadlines.

Very limited prior research has accounted for passengers' personalized requirements on inconvenience factors (e.g. extra in-vehicle travel time, number of shared riders, and additional waiting time) caused by ridesharing in optimizing the vehicle-passenger matching and vehicle routing plan as well as designing customized incentive price simultaneously. The interactive

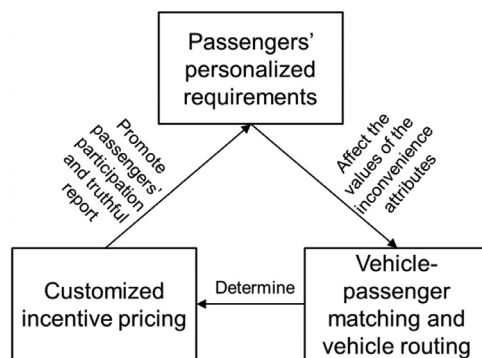


Fig. 1. Relationship among passengers' personalized requirements, optimization of matching and routing plan, and incentive pricing scheme as studied in this paper.

relationship among passengers' personalized requirements, optimization of matching and routing plan, and incentive pricing scheme has not been well studied in the literature.

This paper intends to make the following contributions.

- This paper identifies some of the potential inconvenience factors of scheduled first-mile ridesharing service, including the number of shared riders, extra in-vehicle travel time due to detours, and extra waiting time at the transit hub due to early arrival.
- We present the first work to design an incentive mechanism based on passengers' personalized requirements on these inconvenience attributes by simultaneously optimizing the vehicle-passenger matching and vehicle routing plan and designing a corresponding customized pricing scheme. As Fig. 1 shows, this designed mechanism accounts for the interactive relationship among passengers' personalized requirements, optimization of matching and routing plan, and incentive pricing scheme. Passengers' personalized requirements affect the values of the inconvenience factors in optimizing the matching and routing plan. The customized incentive pricing scheme, which is determined by the matching and routing plan, promotes passengers' participation by offsetting their inconveniences and truthful report of their personalized requirements.
- The incentive mechanism is proved to have the properties of "individual rationality" and "incentive compatibility". It indicates that the mechanism is able to promote rational passengers' participation willingness and also to prevent passengers from manipulating the algorithm.

3. Mechanism design model

This section introduces a ridesharing incentive mechanism based on passengers' personalized requirements. Section 3.1 introduces the problem statement, Section 3.2 analyzes passengers' value and utility when they participate in the service, Section 3.3 clarifies the objective of the proposed mechanism using an optimization model, Section 3.4 introduces how the mechanism is obtained, and Section 3.5 gives the proofs of the properties.

3.1. Problem statement

Passengers can schedule the first-mile ridesharing service in advance. All passengers have the same destination (i.e. the transit hub) to catch their next transit mode (e.g. trains). The service provider, which can be the transit agency or a ridesharing service provider collaborating with the transit agency, has sufficient available vehicles to provide the first-mile accessibility service. Individual passengers may have different preferred times of arrival. Some people may prefer to arrive much earlier than the scheduled train departure time, while others enjoy arriving right on time to catch a train. Thus, our mechanism allows passengers to specify their preferred arrival deadlines at the transit hub. Passengers with close arrival deadlines are likely to share a ride. Vehicles must drive these passengers to the transit hub before the specified deadlines. For example, Mike wants to take the train with the departure time of 9:00 a.m., while the train that John will take departs at 9:10 a.m. Mike wants to arrive at the transit hub on time and thus he specifies 8:50 a.m. as his arrival deadline, while John wants to arrive at the transit hub 25 minutes earlier for breakfast and his arrival deadline is 8:45 a.m. If John and Mike share the ride, the vehicle must arrive at the transit hub before 8:45 a.m.

We use Fig. 2 to demonstrate the operation of the first-mile ridesharing service. The system consolidates passengers' requests with close arrival deadlines. When a passenger schedules the service, he/she is notified of an estimated time window for pickup and a range of trip fare. The time window can be estimated based on passengers' reported arrival deadlines and personalized requirements on extra in-vehicle travel time and extra waiting time at the transit hub. For example, suppose that a passenger's arrival deadline is DL_i , the shortest time for driving this passenger to the transit hub is t_{i0} . Then the latest

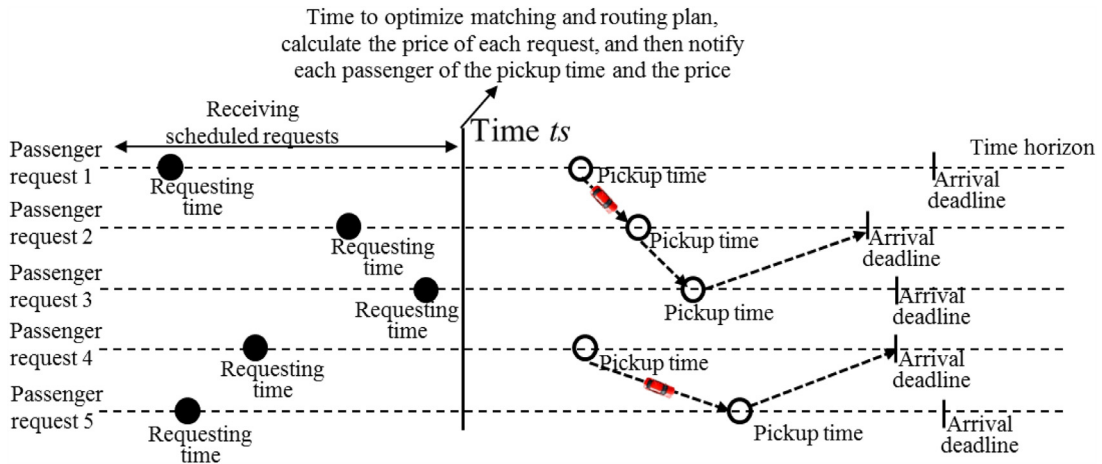


Fig. 2. Operation of the first-mile ridesharing service.

pickup time is $DL_i - t_{i0}$. If this passenger's maximum tolerable extra in-vehicle travel time and extra waiting time at the transit hub are α_i^{IVT} and α_i^{WT} , respectively, then the earliest pickup time is $DL_i - t_{i0} - \alpha_i^{IVT} - \alpha_i^{WT}$. The range of the trip fare can be estimated by historical prices as Uber does. The interface can also show the real-time taxi price in the market. The final price will never exceed this taxi price. When the service is approaching (at time t_s in Fig. 2), the system optimizes the vehicle-passenger matching and vehicle routing plan, and calculates the customized prices. The request processing time point (t_s) should be early enough so that all passengers can be driven to the transit hub before their arrival deadlines. After the requests are processed, each passenger will be notified of the vehicle that will serve him/her, the exact pickup time, and the exact price, which are determined by our mechanism (the matching and routing plan and the pricing scheme). The drivers will be directed to pick up passengers in a specified order and drive them to the transit hub before the earliest arrival deadline.

In addition to the passengers' pickup locations and preferred arrival deadlines, passengers are allowed to report their personalized mobility requirements on different inconvenience factors. In this paper, "inconvenience factors" include: (1) the number of co-riders; (2) extra in-vehicle travel time beyond the direct shipment time due to detour; and (3) extra waiting time at the transit hub due to possible early arrival. Golledge et al. (1994), Ben-Akiva and Lerman (1985), and Arentze (2013) recognized that travelers' choice of transportation mode is influenced not only by price but also by these "inconvenience" attributes. After the system receives the passengers' information, an optimal vehicle-passenger matching and vehicle routing plan is generated based on the personalized requirements. The price is then obtained based on the plan and passengers' reported personalized requirements. Passengers will finally receive a personalized service and customized price. The personalized service is tailored to satisfy passengers' requirements on the inconvenience attributes of the first-mile trip and the customized price is used to incentivize them to participate in the first-mile ridesharing service.

In this paper, it is assumed that each passenger's objective is to maximize their own utility (defined as the difference between the maximum willing-to-pay price and the actual paid price). It is possible that passengers may misreport their requirements on inconvenience factors if lying is more beneficial for them. A desirable property of the pricing mechanism is that expressing the true requirement is the passenger's "best" strategy (i.e. the utility is maximized) regardless of what other passengers report. This property is called "incentive compatibility" in the literature (Myerson, 1979). Passengers' behavioral rationality also implies that if the price is higher than their maximum willing-to-pay price, they are unlikely to participate in the ridesharing service. Thus, another indispensable property, "individual rationality", is that each passenger should always receive non-negative utility with respect to the price charged. This property aims to ultimately incentivize more travelers to participate in the ridesharing service. Moreover, the service provider must receive payment from each passenger (i.e. the price is non-negative). In summary, the proposed mechanism needs to have the three important properties, "incentive compatibility", "individual rationality", and "price non-negativity".

Based on the problem background, we will determine the mechanism, denoted as $M(X, \mathbf{p})$, consisting of a vehicle-passenger matching and vehicle routing plan X and all passengers' customized prices $\mathbf{p} = \{p_1, p_2, \dots, p_n\}$.

The following assumptions are made, in line with the scope of the study.

- (1) We focus on a static case where passengers schedule the service and their information is known in advance. The ridesharing market has placed a demand on pre-scheduled optimization. For example, Uber and Lyft have developed APPs that allow passengers to send pre-scheduled requests for car usage. In this paper, we only optimize the vehicle task execution plan for passengers who send requests before vehicles start to execute the task. In a dynamic scenario, passengers are likely to send requests after the static optimization process is finished, and the system would be able

Table 1
Information for the illustrative example.

Parameters	Passengers		
	John	Peter	Alice
Location coordinates	(2, 2)	(2.6, 2.3)	(3, 2.8)
$V_{\max}^i = 5 + 1.5 \times \max(d_{i0} - 1, 0)$ (taxi price, in dollars)	7.74	8.71	9.66
Time of direct shipment ($t_{i0} = 3 \times d_{i0}$)	8.485	10.414	12.311
Arrival deadlines	13:00 pm	13:10 pm	13:00 pm

Table 2
Passengers' personalized requirements.

Tolerances	Passengers		
	John	Peter	Alice
Maximum in-vehicle travel time (minutes)	10	15	20
Maximum number of shared riders	3	3	2
Maximum waiting time at the transit hub (minutes)	10	15	8

to re-optimize all decisions to accommodate spontaneous demands. The dynamic scenario for spontaneous passengers is beyond the scope of this particular study, but will be considered in our future research.

- (2) The travel time between two locations is assumed to be deterministic. Future research will incorporate travel time reliability in the optimization analysis.
- (3) The fleet size is sufficient to serve all passengers who send requests in advance, and all passengers who send requests will receive the service. The number of passengers in each request does not exceed the seat capacity of a vehicle. Future research will consider fleet shortage given an extraordinarily large ridesharing demand.
- (4) We assume that passengers will not misreport other travel information such as the departure locations, the destination (the transit hub), and the arrival deadlines.

Before we detail the mathematical formulation of the mechanism design, we will use a simple hypothetical example to explain the goal of the research. In this illustrative example, three passengers, named “John”, “Peter”, and “Alice”, in three different locations book the ridesharing service to get to the train station. The transportation cost and the travel time between each two locations as well as the pickup time span at each location are known in advance. For convenience of illustration, the transportation cost (c_{ij}) between two locations is defined as the Euclidean distance (d_{ij}) with one dollar per mile. The travel time (t_{ij}) between two locations is three times the Euclidean distance $t_{ij} = 3d_{ij}$. Note that this illustrative example uses Euclidean distance only for simplification in order to demonstrate how the mechanism is obtained. Our mechanism design model does not assume that the travel distance between two locations should be Euclidean distance. After the vehicle reaches each passenger's location, the vehicle needs some time to pick up the passenger(s). We set the pickup time span as two minutes ($pu_j = 2$) in this example. The coordinate of the transit hub location is set to be (0, 0). The arrival deadlines are determined by the selected train they will catch at the transit hub. We also introduce the taxi service (direct shipment without shared riders) for passengers' alternative first-mile travel mode. The price of the taxi service is \$5 for the first mile and increases \$1.5 per each additional mile. The available information based on the problem setting is listed in Table 1.

Passengers can report their personalized requirements. In this example, we assume that they can report the maximum in-vehicle travel time, the maximum number of co-riders, and the maximum waiting time at the transit hub that they can tolerate. Suppose that their real requirements are given in Table 2.

The problem is how to determine the matching and routing plan and price for each passenger, accounting for passengers' personalized mobility requirements. The proposed mechanism should be able to incentivize passengers to participate in the ridesharing service instead of taking taxi service. Besides, the designed mechanism should incentivize passenger to truthfully report their preferences instead of lying. The results of the mechanism for this example will be displayed in Section 3.4.

3.2. Passengers' value function and utility function

The value function, which reflects passengers' maximum willing-to-pay prices, is used to model passengers' participating willingness considering their personalized requirements on inconvenience attributes. The utility is defined as the net value, which is the maximum willing-to-pay price minus the actual paid price. This paper assumes that rational passengers' objective is to maximize their utilities. Before introducing the value and utility functions, we list the notations in Table 3.

In the context of this research, a passenger's value is defined as the maximum price that he/she is willing to pay, in line with the prior research (Zou et al., 2015; Zhao et al., 2015; Kamar and Horvitz, 2009). This subsection proposes a generalized value function that establishes the relationship between a passenger's value and a given set of inconvenience attributes as well as this passenger's personalized requirement. The personalized requirement, represented by α_i^{NR} , α_i^{IVT} , and α_i^{WT} on the three inconvenience attributes (number of shared riders, extra in-vehicle travel time that exceeds the direct shipment

Table 3
Notations in the value function and utility function.

Notations	Descriptions
i	Index of passenger requests. There can be more than one passenger in each request. For denotation convenience, we let “passenger(s) i ” represent the passenger(s) in request i .
NR_i	Number of co-riders with passenger(s) i .
IVT_i	Passenger(s) i 's in-vehicle travel time.
WT_i	Passenger(s) i 's extra waiting time at the transit hub, i.e. the time interval between the arrival time and the deadline DL_i (see Table 4).
α_i^{NR} , α_i^{IVT} , and α_i^{WT}	Passenger(s) i 's personalized requirement on the number of shared riders, extra in-vehicle travel time that exceeds the direct shipment time, and extra waiting time at the transit hub, respectively. The three parameters are obtained from passengers' reported information.
C_i^{ICN}	Passenger(s) i 's inconvenience cost caused by ridesharing. The inconvenience cost is measured as each passenger's acceptable minimum reduced price with the specific degree of inconvenience factors.
X	A vehicle-passenger matching and vehicle routing plan.
$V_i(X)$	Passenger(s) i 's value gained from the ridesharing service given a plan X . V_i can also be interpreted as the maximum price that this passenger is willing to pay.
V_{\max}^i	The value gained by passenger(s) i when transported from the origin to the transit hub directly without any inconvenience (i.e. $NR_i = 0$, $IVT_i = t_{i0}$, where t_{i0} is passenger(s) i 's direct shipment time, and $WT_i = 0$).
$U_i(X, p_i)$	Passenger(s) i 's utility given a vehicle-passenger matching and vehicle routing plan X and a price p_i .

time due to detour, and extra waiting time at the transit hub due to early arrival) can be in any form, as long as the three parameters α_i^{NR} , α_i^{IVT} , and α_i^{WT} can convey passengers' different tolerances for the inconvenience attributes.

Kamar and Horvitz (2009) proposed a passengers' value function based on inconvenience cost. We incorporate the parameters α_i^{NR} , α_i^{IVT} , and α_i^{WT} as passengers' personalized requirements into the value function.

$$V_i(X) = V_{\max}^i - C_i^{ICN}(NR_i(X), IVT_i(X), WT_i(X), \alpha_i^{NR}, \alpha_i^{IVT}, \alpha_i^{WT}) \tag{1}$$

We list three reasonable assumptions of the parameters in the value function, which are used in the proof of the properties of the proposed mechanism.

- (1) C_i^{ICN} is a monotone increasing function of NR_i , IVT_i , and WT_i . We assume that when people share the trip with more people, stay in the vehicle for a longer time, or wait at the transit hub for extra time, the passengers' inconvenience cost will never decrease.
- (2) We define V_{\max}^i as the price charged by the taxi when this passenger takes this taxi directly to the transit hub without other shared riders. If a passenger participates in the ridesharing service but receives a direct shipment service without other shared riders, the service is treated as taxi service. The maximum willing-to-pay price is equal to the taxi price, because if the price is higher than the taxi price, the customer is unwilling to participate into the ridesharing service and will choose the taxi service. Thus, when $NR_i = 0$, $IVT_i = t_{i0}$, and $WT_i = 0$, the inconvenience cost equals zero. That is

$$C_i^{ICN}(NR_i = 0, IVT_i = t_{i0}, WT_i = 0, \alpha_i^{NR}, \alpha_i^{IVT}, \alpha_i^{WT}) = 0 \tag{2}$$

This assumption is easy to understand because when $NR_i = 0$, $IVT_i = t_{i0}$, and $WT_i = 0$, the service is the same as taxi service—direct shipment for passenger(s) i .

- (3) It is assumed that the taxi always makes a profit. That is, the taxi price is always greater than the transportation cost:

$$V_{\max}^i > c_{i0} \tag{3}$$

A passenger's utility (the difference between the maximum willing-to-pay price and the actual price paid) is given in Formula (4), which is also defined in the literature (Zou et al., 2015; Zhao et al., 2015; Kamar and Horvitz, 2009).

$$U_i(X, p_i) = V_i(X) - p_i \tag{4}$$

We use an illustrative example of the value function for better understanding. This value function will be used in the example in Section 3.4 to illustrate how the mechanism is obtained. In this example, if one passenger shares the ride with others, the maximum willing-to-pay price is set to be the taxi price multiplied by a discount rate (λ_i , here we set the discount rate as $\lambda_i = 0.85$) if the service satisfies the passenger's requirements. Note that the discount rate λ_i can be other values, which is also reported by passengers. If passengers' requirements are not satisfied, they are unwilling to pay anything. Based on this assumption, the value function is defined as:

$$V_i = \begin{cases} V_{\max}^i, & \text{direct shipment (equivalent to taxi service)} \\ 0, & \text{ridesharing, requirements are not satisfied} \\ \lambda_i V_{\max}^i, & \text{ridesharing, requirements are satisfied} \end{cases}$$

The inconvenience cost is thus defined as:

$$C_i^{ICN} = \begin{cases} 0, & \text{direct shipment} \\ V_{\max}^i, & \text{ridesharing, requirements are not satisfied} \\ (1 - \lambda_i) V_{\max}^i, & \text{ridesharing, requirements are satisfied} \end{cases}$$

Table 4

Additional notations of variables and parameters in the optimization model (only for vehicle-passenger matching and vehicle routing optimization).

Sets	
P	Set of passenger requests, $P = \{1, 2, \dots, n\}$
V	Set of vehicles, $V = \{1, 2, \dots, m\}$
H	Set of the transit hub, $H = \{0\}$
Variables	
x_{ijk}	$\begin{cases} 1, & \text{if vehicle } k \text{ travels to location } j \text{ after picking up passenger(s) in location } i \text{ immediately} \\ 0, & \text{otherwise} \end{cases}$
$i \in P, j \in P \cup H, k \in V$	
y_{ik}	$\begin{cases} 1, & \text{if vehicle } k \text{ is dispatched to pick up passenger(s) in location } i \\ 0, & \text{otherwise} \end{cases}$
$i \in P, k \in V$	
$X = \{x_{ijk}, y_{ik} \mid i \in P, j \in P \cup H, k \in V\}$	can represent a vehicle-passenger matching and vehicle routing plan.
w_{ik}	$\begin{cases} 1, & \text{if passenger(s) in location } i \text{ is the first to be picked up by vehicle } k \\ 0, & \text{otherwise} \end{cases}$
$i \in P, k \in V$	
NR_i	Number of co-riders with passenger(s) i .
IVT_i	Passenger(s) i 's in-vehicle travel time.
WT_i	Passenger(s) i 's extra waiting time at the transit hub
C_i^{CN}	Passenger(s) i 's inconvenience cost caused by ridesharing.
Parameters	
np_i	Number of passengers in request i .
DL_i	Passenger(s) i 's preferred deadline before which he/she/they must arrive at the transit hub.
t_{ij}	The travel time from node i to node j , i and $j \in P \cup H$. The pickup time is included in t_{ij} . We assume a triangle inequality assumption $t_{ij} \leq t_{ig} + t_{gj}$ for any i, j , and g , which will be used to guarantee non-negative prices (Section 3.5 Proposition 5). This is a reasonable assumption because the nonstop travel time is unlikely longer than the vehicle's travel time to detour to pick up another passenger plus an additional pickup time.
c_{ij}	The transportation cost from node i to node j , i and $j \in P \cup H$. We assume $c_{ij} \leq c_{ig} + c_{gj}$, for any i, j , and g for the same purpose.
Q	The seat capacity of a vehicle, excluding the driver.

Let us return to the example in Section 3.1. John's value function is as follows:

$$V_{\text{John}} = \begin{cases} 7.74, & IVT_{\text{John}} = t_{i0}, NR_{\text{John}} = 0, WT_{\text{John}} = 0 \\ 0, & IVT_{\text{John}} > 10, NR_{\text{John}} > 3, \text{ or } WT_{\text{John}} > 10 \\ \lambda_i \times 7.74, & \text{otherwise} \end{cases}$$

Note that the example above is just an illustrative example. The value function can take a generalized form that is adaptable to any reasonable scenario. Developing specific value functions and designing an interface that allows users to report their requirements are beyond the scope of this paper, but will be considered in future research.

3.3. Optimization of vehicle-passenger matching and routing plan

We consider the ridesharing service provider (the agency) and passengers (the users) as a system to optimize the vehicle-passenger matching and routing plan. The agency and the users are two indispensable components of a system, and both the agency cost and the user cost are often considered collaboratively in the literature (Kim et al., 2015; Hajibabai et al., 2014; Amirgholy and Gonzales, 2016). The objective of the proposed mechanism is to minimize the agency's transportation cost (e.g. vehicle dispatch cost, energy consumption cost, driver labor cost, and emissions) and the users' inconvenience cost caused by ridesharing associated with their personalized requirements. This formulates an optimization problem to determine an optimal vehicle-passenger matching and vehicle routing plan.

The problem can be formulated as the following model (denoted as IP). For the notations, please refer to Tables 3 and 4.

$$Z = \min \sum_{i \in P} C_i^{CN} (NR_i, IVT_i, WT_i, \alpha_i^{NR}, \alpha_i^{IVT}, \alpha_i^{WT}) + TC(X) \tag{5}$$

where $TC(X)$ is the transportation cost of the vehicle-passenger matching and vehicle routing plan: $TC(X) = \sum_{k \in V} \sum_{i \in P} \sum_{j \in P \cup H \setminus i} x_{ijk} c_{ij}$

Subject to

$$\sum_{k \in V} y_{ik} = 1, \text{ for all } i \in P \tag{6}$$

$$\sum_{i \in P} y_{ik} np_i \leq Q, \text{ for all } k \in V \tag{7}$$

$$w_{ik} + \sum_{i \in P \setminus j} x_{ijk} = y_{jk}, \text{ for all } k \in V, j \in P \tag{8}$$

$$\sum_{j \in P \cup H \setminus i} x_{ijk} = y_{ik}, \text{ for all } k \in V, i \in P \tag{9}$$

$$\sum_{i \in P} w_{ik} \leq 1, \text{ for all } k \in V \tag{10}$$

$$IVT_i = \sum_{k \in V} \sum_{j \in H \cup P \setminus i} x_{ijk} (IVT_j + t_{ij}), \text{ for all } i \in P \tag{11}$$

$$IVT_i \geq 0, \text{ for all } i \in P \tag{12}$$

$$WT_i = DL_i - \min_{j \in P} \left\{ M \left(1 - \sum_{k \in V} y_{jk} y_{ik} \right) + DL_j \right\}, \text{ for all } i \in P \tag{13}$$

$$NR_i = \sum_{j \in P \setminus i} \sum_{k \in V} y_{ik} y_{jk} n p_j, \text{ for all } i \in P \tag{14}$$

$$x_{ijk}, y_{ik}, w_{ik} \in \{0, 1\}, \text{ for all } i, j \in P \cup H, k \in V \tag{15}$$

Formula (5) is the objective function that minimizes both the passengers' inconvenience cost and the agency's transportation cost. One passenger's inconvenience cost is a function of the number of co-riders, in-vehicle travel time, and extra waiting time at the transit hub. Formula (6) ensures that all passengers will be picked up by one vehicle and only be served once. Formula (7) represents that the maximum capacity of each vehicle should not be exceeded. Formulas (8) and (9) ensure the balanced flow from and to each passenger location. Formula (10) ensures that each vehicle can only be dispatched once at most. Formula (11) gets all passengers' in-vehicle travel times. Formula (12) is to ensure the non-negativity of all passengers' in-vehicle travel times. Formulas (13) and (14) get all passengers' extra waiting times at the transit hub and the number of shared riders, respectively. Formula (15) signifies that x_{ijk} , y_{ik} , and w_{ik} are binary variables.

We do not use constraints to formulate passengers' requirements because we already use the inconvenience cost function to represent the passengers' requirements. Thus, adding constraints to represent passengers' requirements is redundant and unnecessary. We use the example in Section 3.2 to demonstrate this.

Proposition 1. *Adding inconvenience cost function into the objective function can ensure that passenger(s) i's (for all $i \in P$) personalized requirement can be always satisfied.*

Proof. Suppose that X^* is the optimal solution of model IP and passenger(s) i 's requirement is not satisfied given the optimal matching and routing plan X^* . Thus, passenger(s) i 's inconvenience cost is $C_i^{ICN}(X^*) = V_{\max}^i$. Let $Z(X^*)$ represent the objective function value of model IP (Formula 5). If passenger(s) i does not participate in the first-mile ridesharing service, then the optimal objective function value is assumed to be Z_i . It is easy to understand that $Z_i \leq Z(X^*) - C_i^{ICN}(X^*) = Z(X^*) - V_{\max}^i$, because extra transportation cost is needed for a vehicle to serve passenger(s) i . Now consider a solution X_i in which passenger(s) i is shipped to the transit hub directly without shared riders, and the matching and routing plan is optimized for other passengers. Thus, $Z(X_i) = Z_i + C_i^{ICN}(X_i) + c_{i0}$. Since passenger(s) i is shipped to the transit hub directly without shared riders in X_i , passenger(s) i does not have the inconvenience cost ($C_i^{ICN}(X_i) = 0$), and thus $Z(X_i) = Z_i + c_{i0}$. $Z_i = Z(X_i) - c_{i0} \leq Z(X^*) - V_{\max}^i$, then $Z(X^*) - Z(X_i) \geq V_{\max}^i - c_{i0}$. Based on Formula (3), $V_{\max}^i - c_{i0} > 0$. Thus $Z(X^*) > Z(X_i)$. Since X_i is a feasible solution of model IP , the optimality of solution X^* is violated. Thus, passengers' requirements can be always satisfied in the optimal solution X^* of model IP . \square

3.4. Customized incentive pricing scheme

This subsection firstly introduces the basic idea of designing an individual rational and incentive compatible pricing strategy. Then, we detail the design of the pricing scheme for the particular application in first-mile ridesharing. Subsequently, we use a simple example to show how the price is obtained. The theoretical proofs of the properties, individual rationality, incentive compatibility, and price non-negativity are given in Section 3.5.

The pricing framework is calculated by designing and solving a series of models, including one model IP_0 and n models IP_g (for all $g \in P$). Model IP_0 should be equivalent to the original model IP proposed in Section 3.3. Each model IP_g is used to calculate the price only and does not have practical meaning. Both models IP_0 and IP_g have maximizing objective functions. Then the pricing scheme is given by

$$p_g = g(X^{IP_g^*}) - (f(X^{IP_0^*}) - V_g(X^{IP_0^*})) \tag{16}$$

Table 5
Mathematical models for obtainment of the mechanism.

Model denotations	Objective functions	Constraints	Optimal solution	Optimal objective function value
IP_0	$f(X)$: Formula (17)	CS_{IP_0} Formula (18)	$X^{IP_0*} = \{x_{ijk}^{IP_0*}, y_{ik}^{IP_0*}\}$	$Z_{IP_0}^*$
IP_g for all $g \in P$	$g(X)$: Formula (17)	CS_{IP_g} Formulas (18, 19)	$X^{IP_g*} = \{x_{ijk}^{IP_g*}, y_{ik}^{IP_g*}\}$	$Z_{IP_g}^*$

X^{IP_0*} is the optimal solution of model IP_0 with the maximizing objective function $f(\cdot)$, which includes the summation of all passengers' values.

$$f(X) = \sum_{i \in P} V_i(X) + h(X) \tag{16a}$$

where $h(X)$ is used to make the model IP_0 equivalent to the original model IP proposed in Section 3.3.

X^{IP_g*} is the optimal solution of model IP_g , and $g(\cdot)$ is the maximizing objective function of the model.

This pricing scheme makes the mechanism "individual rational" if the following condition is always satisfied

$$g(X^{IP_g*}) \leq f(X^{IP_0*}) \tag{16b}$$

This is because passenger(s) g 's utility is $U_g = V_g(X^{IP_0*}) - p_g = f(X^{IP_0*}) - g(X^{IP_g*}) \geq 0$, if the condition above is satisfied. A direct way of satisfying this condition is to design the model IP_g that makes the objective function $g(X)$ identical with $f(X)$ and to let the feasible regions of models IP_g (for each g) be included in the feasible region of model IP_0 . That is

$$g(X) = f(X) \tag{16c}$$

$$CS_{IP_g} \subseteq CS_{IP_0} \tag{16d}$$

where CS_{IP_g} and CS_{IP_0} are the feasible regions of models IP_g and IP_0 , respectively.

If model IP_g is independent of passenger(s) g 's report, then the mechanism is "incentive compatible".

If passenger(s) g misreports the requirement, then we assume that the optimal solution of model IP_0 changes from X^{IP_0*} to Y^{IP_0*} . $g(X^{IP_g*})$ remains constant because $g(X^{IP_g*})$ is independent of passenger(s) g 's report. Then $f(X^{IP_0*})$ changes to

$$f'(Y^{IP_0*}) = \sum_{i \in P, i \neq g} V_i(Y^{IP_0*}) + V'_g(Y^{IP_0*}) + h(Y^{IP_0*}) \tag{16e}$$

Then, the price becomes

$$p'_g = g(X^{IP_g*}) - (f'(Y^{IP_0*}) - V'_g(Y^{IP_0*})) = g(X^{IP_g*}) - \left(\sum_{i \in P, i \neq g} V_i(Y^{IP_0*}) + h(Y^{IP_0*}) \right) \tag{16f}$$

Then passenger(s) g 's utility becomes

$$U'_g = V_g(Y^{IP_0*}) - p'_g = \left(\sum_{i \in P} V_i(Y^{IP_0*}) + h(Y^{IP_0*}) \right) - g(X^{IP_g*}) = f(Y^{IP_0*}) - g(X^{IP_g*}) \tag{16g}$$

Y^{IP_0*} may no longer be optimal for model IP_0 , indicating that the objective function of model IP_0 , $f(\cdot)$, will suffer from a decrease caused by her misreporting. Thus, her utility $U_g = f(Y^{IP_0*}) - g(X^{IP_g*})$ will decrease as well if she misreports her personalized requirement. Therefore, truthful reporting is passengers' optimal strategy.

The following content of this section details the design of pricing strategy in Formula (16). The designed models IP_0 and IP_g (for all $g \in P$) are summarized in Table 5.

Model IP_0 :

Objective function:

$$\max Z_0(X) = \sum_{i \in P} V_i(X) - TC(X) \tag{17}$$

where $TC(X)$ is the transportation cost of the routing plan X .

$$TC(X) = \sum_{k \in V} \sum_{i \in P} \sum_{j \in P \cup H \setminus i} x_{ijk} c_{ij}$$

Constraints:

$$X \in CS_{IP_0} \tag{18}$$

The constraint set CS_{IP_0} consists of Formulas (6)–(15).

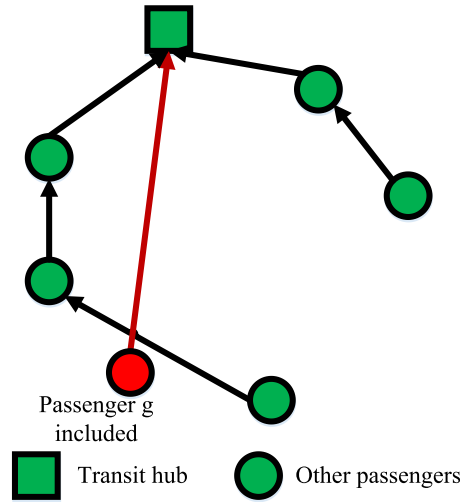


Fig. 3. An optimal solution of IP_g .

IP_0 is mathematically equivalent to the original optimization model (IP) in Section 3.3. First, IP_0 and IP have identical constraints. Second, the objective functions of the two models are equivalent implied by Formulas (1), (5) and (17). Thus, $X^{IP_0^*}$ is also the optimal solution of the original optimization model (IP) in Section 3.3.

Models IP_g (along with IP_0 , IP_g is to calculate each passenger’s price if he/she/they participates in the first-mile ridesharing service):

Objective function: Formula (17).

Constraints (CS_{IP_g}): Formulas (18) and (19)

$$NR_g = 0 \tag{19}$$

Each model optimizes all passengers’ values minus the transportation cost in the system given that passenger(s) g is transported to the transit hub directly without any shared riders (see Fig. 3).

All passengers’ prices are $\mathbf{p} = \{p_1, p_2, \dots, p_n\}$, in which each price is calculated by

$$p_g = Z_{IP_g}^* - (Z_{IP_0}^* - V_g(X^{IP_0^*})) \tag{20}$$

Model IP_0 and IP_g have an identical objective function (Formula 17) and the feasible region of model IP_g is included in the feasible region of model IP_0 because model IP_g has an additional constraint (Formula 19) compared with model IP_0 . Thus, the mechanism is “individual rational” based on Formulas (16c) and (16d). Moreover, the optimal solution of model IP_g is independent of passenger(s) g ’s report of the parameters of α_g^{NR} , α_g^{IVT} , and α_g^{WT} because passenger(s) g ’s inconvenience cost is zero and the value is a constant (V_{\max}^g) if the passenger(s) is transported to the transit hub directly without shared riders, no matter what values of α_g^{NR} , α_g^{IVT} , and α_g^{WT} the passenger(s) reports. This can ensure “incentive compatibility” based on Formulas (16e)–(16g). In addition, the mechanism has another important property, “price non-negativity”, which ensures that the service provider can receive revenue from passengers. The detailed proofs of these three properties are given in Section 3.5.

The mechanism can be denoted as $M(X^{IP_0^*}, \mathbf{p})$, and Algorithm 1 shows how the mechanism is obtained.

Let us return to the simple example proposed in Section 3.1 to show how the mechanism is obtained. The three passengers John, Peter, and Alice are numbered as “1”, “2”, and “3”, and the transit hub is numbered as “0”. We use the value function in the example proposed in Section 3.2. We firstly optimize the model IP_0 to get the optimal solution $X^{IP_0^*}$ of model IP_0 , which is a vehicle-passenger matching and vehicle routing plan: “Alice–Peter–John–Transit hub” (3-2-1-0, Fig. 4). The total transportation cost of this routing plan ($TC(X^{IP_0^*})$) is 4.140 dollars. The optimization results are summarized in Table 6.

Then we consider the three models IP_1 , IP_2 , and IP_3 (IP_g , $g = 1, 2$, and 3. John: 1, Peter: 2, Alice: 3). The optimization results are listed in Table 7.

Take John as an example to show how his price is calculated. John’s price is calculated by Formula (20):

$$\begin{aligned} p_1 &= (V_1(X^{IP_1^*}) + V_2(X^{IP_1^*}) + V_3(X^{IP_1^*}) - TC(X^{IP_1^*})) - (V_2(X^{IP_0^*}) + V_3(X^{IP_0^*}) - TC(X^{IP_0^*})) \\ &= (7.743 + 7.401 + 8.207 - 6.940) - (7.401 + 8.207 - 4.140) = 4.94 \text{ (dollars)}. \end{aligned}$$

Others’ prices are calculated in the same method. The result of the mechanism is given in Table 8.

We then take Alice as an example to show three possible strategies: 1) taking a taxi to achieve direct shipment; 2) participating in the ridesharing service and truthfully reporting her requirement ($\alpha_i^{IVT} = 20$, $\alpha_i^{NR} = 2$, and $\alpha_i^{WT} = 8$, the maximum in-vehicle travel time, the maximum number of co-riders, and the maximum extra waiting time at the transit hub that the

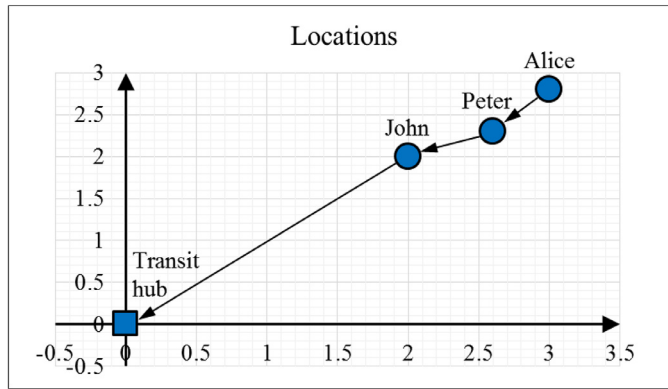


Fig. 4. Optimal routing plan of the example.

Table 6
Optimization results of IP_0 .

Optimization results	Passengers		
	John	Peter	Alice
Total travel time (minutes)	8.5	12.5	16.4
Extra waiting time at the transit hub (minutes)	0	10	0
Number of shared riders	2	2	2
Values $V_i(X^{IP_0^*})$ (dollars)	6.58	7.40	8.21

Notation: $V_i(X^{IP_0^*})$, passenger i 's value given the optimal plan $X^{IP_0^*}$, i.e. the maximum willing-to-pay price.

Table 7
Optimization results of IP_g .

	Models (IP_g)								
	IP_1			IP_2			IP_3		
Optimal solution	$X^{IP_1^*}(3-2-0, 1-0)$			$X^{IP_2^*}(3-1-0, 2-0)$			$X^{IP_3^*}(2-1-0, 3-0)$		
Total Transportation cost ($TC(X^{IP_g^*})$)	6.94			7.58			7.60		
Passenger indexes (i)	1	2	3	1	2	3	1	2	3
Travel time (minutes)	8.49	10.41	14.34	8.49	10.41	14.33	8.49	12.50	12.31
Waiting time at transit hub (minutes)	0	10	0	0	0	0	0	10	0
Number of shared riders	0	1	1	1	0	1	1	1	0
$V_i(X^{IP_g^*})$ (dollars)	7.74	7.40	8.21	6.58	8.71	8.21	6.58	7.40	9.66

Notation: $V_i(X^{IP_g^*})$, passenger i 's ($i = 1, 2, 3$) value (i.e. the maximum willing-to-pay price) given the optimal solution ($X^{IP_g^*}$) of the model IP_g . (3-2-0, 1-0), a vehicle-passenger matching and vehicle routing plan, in which two vehicles are used (Vehicle 1: 3-2-0, Vehicle 2: 1-0).

Table 8
The result of the customized pricing mechanism.

Optimal routing plan	Alice->Peter->John->the transit hub		
	John	Peter	Alice
Passengers			
Taxi price (V_{max}^i , in dollars)	7.74	8.71	9.65
Maximum willing-to-pay price ($\lambda_i \times V_{max}^i$, in dollars)	6.58	7.40	8.21
Actual payment (dollars)	4.94	5.27	6.21
Utility (dollars)	1.64	2.13	2.00

passenger can tolerate are 20, 2, and 8 minutes, respectively, as shown in Table 2); and 3) participating in the ridesharing service and misreporting her requirement ($\alpha_i^{VT} = 15$, which is a misreported value, $\alpha_i^{NR} = 2$, and $\alpha_i^{WT} = 8$). Table 9 shows the results of the three strategies. If she takes the taxi, her utility is “0” as she pays the maximum willing-to-pay price. If she uses the ridesharing service and truthfully reports the requirement, the price is only 6.21 dollars and her utility is 2 dollars. However, if she misreports her requirement, the price increases to 7.85 and her utility decreases to 0.36. This table demonstrates that participating in the ridesharing service and telling the truth is the optimal strategy for this passenger (the bold number “2.00” is the maximum utility).

Table 9
Alice's three strategies and the corresponding results.

Alice's service attributes	Strategies		
	Direct shipment (taking taxi)	Ridesharing, telling the truth	Ridesharing, misreporting
Optimal routing plan generated by the system	(3-0, 2-0, 1-0)	(3-2-1-0)	(3-2-0, 1-0)
Actual value (dollars)	9.66	8.21	8.21
Price (dollars)	9.66	6.21	7.85
Utility (dollars)	0	2.00	0.36

3.5. Theoretical analysis

This subsection presents the mathematical description of the three properties “individual rationality”, “incentive compatibility”, and “price non-negativity”, and gives mathematical proofs.

Proposition 2. *Individual rationality.* As long as a passenger participates in the service system, the mechanism $M(X^{IP_0^*}, \mathbf{p})$ ensures that each passenger's utility ($U_g(X^{IP_0^*}, p_g)$) received from the ridesharing service is always non-negative.

$$U_g(X^{IP_0^*}, p_g) = V_g(X^{IP_0^*}) - p_g \geq 0, \text{ for any } g \in P \tag{21}$$

Proof.

$$\begin{aligned} U_g(X^{IP_0^*}, p_g) &= V_g(X^{IP_0^*}) - p_g \\ &= V_g(X^{IP_0^*}) - Z_{IP_g}^* + (Z_{IP_0}^* - V_g(X^{IP_0^*})) \\ &= Z_{IP_0}^* - Z_{IP_g}^* \\ &= Z_{IP_0}^* - Z_0(X^{IP_g^*}) \end{aligned}$$

The first part of the formula above is the optimal objective function value of IP_0 . $X^{IP_g^*}$ is a feasible solution of IP_0 , and thus the second part of the formula is not necessarily the optimal objective function value of IP_0 . Thus,

$$U_g(X^{IP_0^*}, p_g) = Z_{IP_0}^* - Z_0(X^{IP_g^*}) \geq 0$$

Proposition 3. *Incentive compatibility.* Telling the truth is always the optimal reporting strategy for each passenger who participates in the service under the mechanism $M(X^{IP_0^*}, \mathbf{p})$ regardless of other passengers' reporting strategies (Nisan et al., 2007).

Proof. We assume that passenger(s) g misreports the parameters (personalized requirements, denoted as θ_g) in the value function. In the example in Section 3.2, $\theta_g = \{\alpha_g^{NR}, \alpha_g^{IVT}, \alpha_g^{WT}, \lambda_g\}$.

We define $V_g'(X) = V_{\max}^g - C^{ICN}(NR_g(X), IVT_g(X), WT_g(X), \theta_g')$, where θ_g' is the set of passenger(s) g 's misreported values in θ_g .

The optimization problem IP_0 becomes IP_0' :

$$Z_{IP_0'}^* = \max Z_0'(X) = \sum_{i \in P, i \neq g} V_i(X) + V_g'(X) - TC(X), \text{ s.t. } X \in CS_{IP_0}$$

Note that model IP_0' uses all passengers' reported personalized requirements as input data, in which passenger(s) g 's personalized requirement is misreported. Other passengers' values ($V_i(X)$, for all $i \neq g$) are calculated based on their reported personalized requirements regardless of whether these passengers' reports are truthful or not. The only difference of IP_0 from IP_0' is that model IP_0 uses passenger(s) g 's truthful report as an input data. We assume that $X^{IP_0^*}$ is the optimal solution of IP_0' . Optimization model IP_g does not change, because problem IP_g is independent of passenger(s) g 's report. More precisely, passenger(s) g 's value always equals V_{\max}^g (implied from Formulas 1 and 2) because the passenger(s) is directly transported to the transit hub without shared riders in IP_g (see Fig. 3).

Then, the price charged for passenger(s) g is:

$$p_g' = Z_{IP_g}^* - \left(Z_{IP_0'}^* - V_g'(X^{IP_0^*}) \right)$$

The utility that passenger(s) g can receive is:

$$\begin{aligned} U_g(X^{IP_0^*}, p_g') &= V_g(X^{IP_0^*}) - p_g' \\ &= V_g(X^{IP_0^*}) - \left(Z_{IP_g}^* - \left(Z_{IP_0'}^* - V_g'(X^{IP_0^*}) \right) \right) \\ &= V_g(X^{IP_0^*}) - \left(Z_{IP_g}^* - \left(\sum_{i \in P, i \neq g} V_i(X^{IP_0^*}) + V_g'(X^{IP_0^*}) - TC(X^{IP_0^*}) - V_g'(X^{IP_0^*}) \right) \right) \\ &= \sum_{i \in P} V_i(X^{IP_0^*}) - TC(X^{IP_0^*}) - Z_{IP_g}^* \\ &= Z_0(X^{IP_0^*}) - Z_{IP_g}^* \end{aligned}$$

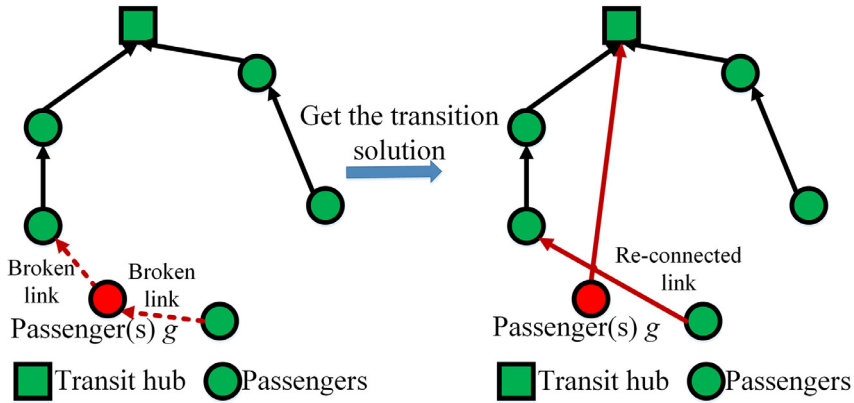


Fig. 5. Example of the transition solution obtainment.

$X^{IP_0'}$ is not necessarily the optimal solution of IP_0 , thus

$$Z_0(X^{IP_0'}) \leq Z_0(X^{IP_0^*})$$

Accordingly, we have

$$U_g(X^{IP_0'}, p'_g) = Z_0(X^{IP_0'}) - Z_{IP_g}^* \leq Z_0(X^{IP_0^*}) - Z_{IP_g}^* = U_g(X^{IP_0^*}, p_g)$$

where $U_g(X^{IP_0^*}, p_g)$ is passenger(s) g 's utility and $X^{IP_0^*}$ is the optimal solution of model IP_0 when he reports the true values in θ_g . $X^{IP_0'}$ and $X^{IP_0^*}$ are respectively the optimal solutions of models IP_0' and IP_0 , regardless of other passengers' reporting strategies. This indicates that telling the truth is always the best strategy for each passenger regardless of other passengers' reporting strategies. □

As passengers consider adopting different strategies, it becomes a game in which they decide whether to take a ridesharing service or taxi service and choose to truthfully report or misreport their personalized requirements. The properties of “individual rationality” and “incentive compatibility” of the designed mechanism ensure a Nash Equilibrium state of the game in which all passengers take ridesharing service and truthfully report their personalized requirements. This indicates that taking ridesharing service and truthfully reporting is each passenger's dominant strategy.

Then, we introduce the definition of “transition solution” and analyze its property (Proposition 4). The definition of “transition solution” will be used to demonstrate that the mechanism has the property of “price non-negativity” (Section 3.5 Proposition 5).

Definition 1. $Y_g = TRS_g(X)$ is the g th transition solution from a feasible solution X of the model IP_0 to the corresponding feasible solution Y_g of the model IP_g if the transition process is given by Algorithm 2.

Fig. 5 shows an example of transition solution generation. Passenger(s) g goes to the transit hub directly without any other shared passengers, and the broken links are re-connected.

Proposition 4. For any passenger(s) i , $V_i(Y_g) \geq V_i(X)$ for any solution X , where $Y_g = TRS_g(X)$ for any $g \in P$.

This proposition will be used in the proof of the “price non-negativity” proposition (Section 3.5 Proposition 5).

Proof. If $i = g$, then $V_i(Y_g) = V_{\max}^i$, and thus $V_i(Y_g) \geq V_i(X)$.

If passengers in requests i and g are served by the same vehicle, we have $IVT_i(Y_g) \leq IVT_i(X)$, $NR_i(Y_g) \leq NR_i(X)$, and $WT_i(Y_g) \leq WT_i(X)$. Since the passengers' value function is a monotone decreasing function of NR_i , IVT_i , and WT_i , we have $V_i(Y_g) \geq V_i(X)$.

If passenger(s) i and g are served by different vehicles, $V_i(Y_g) = V_i(X)$, because passenger(s) i 's matching and routing plan is the same in Y_g as in X .

Thus, for any passenger(s) i , we have $V_i(Y_g) \geq V_i(X)$ for any transition solution g . □

Proposition 5. Price non-negativity. If two preconditions are satisfied: 1) the transportation cost and travel time between two locations comply with the triangle inequality $c_{ij} \leq c_{ig} + c_{gj}$ and $t_{ij} \leq t_{ig} + t_{gj}$ for any i, j , and g ; and 2) $V_{\max}^i > c_{i0}$ (Formula 3), the service provider can always receive revenue from each passenger under the mechanism $M(X^{IP_0^*}, \mathbf{p})$.

$$p_g = Z_{IP_g}^* - (Z_{IP_0}^* - V_g(X^{IP_0^*})) \geq 0 \tag{22}$$

Table 10
Addresses of the ten selected locations.

Passengers	Addresses	Passengers	Addresses
1	458 Ralph St, Somerset, NJ	6	Rockefeller Road, Piscataway Township, NJ
2	16 King Rd, Somerset, NJ	7	227 Hilton St, Highland Park, NJ
3	58 Arden St, Somerset, NJ	8	121 S 11th Ave, Highland Park, NJ
4	235 Hampshire Court, Piscataway Township, NJ	9	109 S 8th Ave, Highland Park, NJ
5	375 Lancaster Ct, Piscataway Township, NJ	10	219 S 7th Ave, Highland Park, NJ

Let $Y_g = TRS_g(X^{IP_0^*}) = \{x_{ijk}^g, y_{ik}^g | i \in P, j \in P \cup H, k \in V\}$ (see Definition 1). Since Y_g is a feasible solution of IP_g and $X^{IP_0^*}$ is the optimal solution of IP_g , we have $Z_0(X^{IP_0^*}) \geq Z_0(Y_g)$. Thus,

$$p_g = Z_0(X^{IP_0^*}) - (Z_{IP_0}^* - V_g(X^{IP_0^*})) \geq Z_0(Y_g) - (Z_{IP_0}^* - V_g(X^{IP_0^*}))$$

Since in solution Y_g passenger(s) g is transported to transit hub without shared riders, $V_g(Y_g) = V_{\max}^g$. Thus

$$Z_0(Y_g) = \sum_{i \in P \setminus g} V_i(Y_g) - \sum_{k \in V} \sum_{i \in P \setminus g} \sum_{j \in P \cup H \setminus i, g} x_{ijk}^g c_{ij} + (V_{\max}^g - c_{g0})$$

From Formula (3), we have

$$Z_0(Y_g) > \sum_{i \in P \setminus g} V_i(Y_g) - \sum_{k \in V} \sum_{i \in P \setminus g} \sum_{j \in P \cup H \setminus i, g} x_{ijk}^g c_{ij}.$$

From Proposition 4, we have

$$\sum_{i \in P \setminus g} V_i(Y_g) \geq \sum_{i \in P \setminus g} V_i(X^{IP_0^*})$$

Thus

$$Z_0(Y_g) > \sum_{i \in P \setminus g} V_i(X^{IP_0^*}) - \sum_{k \in V} \sum_{i \in P \setminus g} \sum_{j \in P \cup H \setminus i, g} x_{ijk}^g c_{ij}$$

$\sum_{k \in V} \sum_{i \in P \setminus g} \sum_{j \in P \cup H \setminus i, g} x_{ijk}^g c_{ij}$ is the transportation cost excluding the transportation cost that is related to passenger(s) g in solution Y_g . It is easily proved smaller than or equal to the total transportation cost in solution $X^{IP_0^*}$ ($\sum_{k \in V} \sum_{i \in P} \sum_{j \in P \cup H \setminus i} x_{ijk}^{IP_0^*} c_{ij}$) because of the triangle equality. Thus

$$Z_0(Y_g) > \sum_{i \in P, i \neq g} V_i(X^{IP_0^*}) - \sum_{k \in V} \sum_{i \in P} \sum_{j \in P \cup H \setminus i} x_{ijk}^{IP_0^*} c_{ij} = Z_{IP_0}^* - V_g(X^{IP_0^*})$$

Thus

$$p_g > Z_0(Y_g) - (Z_{IP_0}^* - V_g(X^{IP_0^*})) \geq 0$$

4. Case study

4.1. Input setting

This section presents a case study to illustrate the results of the designed mechanism and its theoretical properties. In the following case, we select ten locations near the New Brunswick Train Station (New Jersey, in the United States) on Google Maps. The addresses of the ten locations are listed in Table 10 and are identified in Fig. 6 on the map. The travel times between two locations are estimated by Google Maps at 12:30 p.m. on July 13, 2017. The travel distance between two locations is obtained based on the actual routes using the information from Google Maps. For convenience of clarification, the transportation cost is set to be proportional to the travel distance. The taxi price (V_{\max}^i) is \$5 for the first mile and \$1.5 for each additional mile, $V_{\max}^i = 5 + 1.5 \times \max(d_{i0} - 1, 0)$. Each location has one passenger sending the request for the service. We assume that each passenger catches one of the three trains at New Brunswick Station. Passengers' train schedule information is listed in Table 11. In our case study, for simplicity and ease of illustration, all the passengers' preferred arrival deadlines are set to be ten minutes before their train departure times. Our model can also handle the problems when their preferred arrival deadlines are different. A fleet of cars with a seat capacity of "4" will be dispatched to pick up all the passengers and transport them to the transit hub before the specified deadlines.

The case study uses two types of value functions and passengers' report methods in order to show that the generalized mechanism can be adapted to different scenarios. In the first scenario, passengers can report the maximum extra in-vehicle

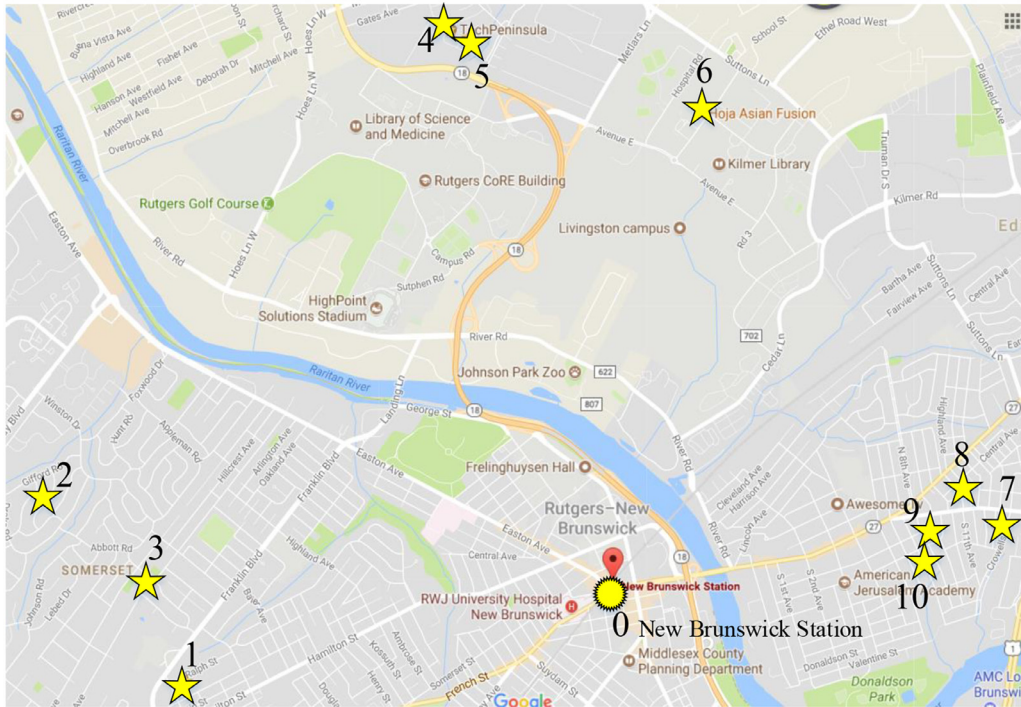


Fig. 6. Selected locations near New Brunswick Station.

Table 11
Trains in New Brunswick Station selected by the ten passengers.

Passengers indexes (<i>i</i>)	Train numbers	Train departure times	Passengers indexes (<i>i</i>)	Train numbers	Train departure times
1	Q3846	1:20 pm	6	Q3846	1:20 pm
2	Q3846	1:20 pm	7	Q3843	1:35 pm
3	Q3848	1:36 pm	8	Q3843	1:35 pm
4	Q3848	1:36 pm	9	Q3843	1:35 pm
5	Q3843	1:35 pm	10	Q3848	1:36 pm

Table 12
Passengers' personalized requirements in the first scenario.

Personalized requirements	Passenger indexes									
	1	2	3	4	5	6	7	8	9	10
α_i^{NR}	3	3	3	4	4	3	4	4	4	4
α_i^{IVT} (minutes)	10	15	15	10	6	8	7	15	10	10
α_i^{WT} (minutes)	20	20	5	10	10	20	10	5	10	15

α_i^{NR} : the maximum number of shared riders that the passenger *i* can tolerate.
 α_i^{IVT} : the maximum extra in-vehicle travel time that the passenger *i* can tolerate.
 α_i^{WT} : the maximum extra waiting time at the transit hub that the passenger *i* can tolerate.

travel time, maximum number of shared riders, and maximum extra waiting time at the transit hub (see Table 12), as shown in the example in Section 3. Passengers' value function is as that of the example in Section 3.2:

$$V_i = \begin{cases} V_{\max}^i, & \text{direct shipment} \\ 0, & \text{ridesharing, requirements are not satisfied} \\ \lambda_i V_{\max}^i, & \text{ridesharing, requirements are satisfied} \end{cases}$$

Passengers' reporting methods and the value function are only used for illustration, and the method can be adapted to any specific form.

In the first scenario, passengers can directly report their personalized requirements. The interactive system is straightforward for users to manipulate. However, the system has one limitation: as long as one passenger's requirements are satisfied, the value (maximum willing-to-pay price) is assumed to be a constant, $\lambda_i V_{\max}^i$, even though the service has different degrees

Table 13
Passengers' personalized requirements in the second scenario.

Personalized requirements	Passenger indexes									
	1	2	3	4	5	6	7	8	9	10
α_i^{NR}	0.12	0.29	0.41	0.35	0.18	0	0.10	1.00	0.19	0.20
α_i^{IVT}	0.30	0.40	0.51	0.44	0.82	1.66	0.62	1.89	0.32	1.20
α_i^{WT}	0.10	0.10	1.52	1.79	0.83	0.03	0.76	0.88	1.25	2.00

α_i^{NR} (dollars per co-rider): reduction rate of maximum willing-to-pay price in terms of the number of co-riders.
 α_i^{IVT} (dollars every 5 minutes): reduction rate of maximum willing-to-pay price in terms of the extra in-vehicle travel time.
 α_i^{WT} (dollars every 5 minutes): reduction rate of maximum willing-to-pay price in terms of the extra waiting time at the transit hub.

Table 14
Results of the mechanism in the two scenarios.

Results	(a) Results of the mechanism in the first scenario									
	Passenger indexes (<i>i</i>)									
	1	2	3	4	5	6	7	8	9	10
V_{\max}^i (in dollars)	7.25	8.60	8.15	10.55	10.70	8.30	6.20	6.05	5.75	5.90
V_i (in dollars)	6.16	7.31	6.93	8.97	9.10	7.06	5.27	5.14	4.89	5.02
p_i (in dollars)	4.95	6.01	5.63	6.05	6.20	5.40	4.80	4.55	4.35	4.50
U_i (in dollars)	1.21	1.30	1.30	2.92	2.90	1.66	0.47	0.59	0.54	0.52
Results	(b) Results of the mechanism in the second scenario									
	Passenger indexes (<i>i</i>)									
	1	2	3	4	5	6	7	8	9	10
V_i (in dollars)	6.69	6.98	6.82	9.15	9.52	8.20	5.85	5.05	5.25	5.70
p_i (in dollars)	6.56	6.53	6.14	6.40	6.81	6.96	5.70	4.96	5.02	5.49
U_i (in dollars)	0.13	0.45	0.68	2.75	2.71	1.24	0.15	0.09	0.23	0.21

V_{\max}^i : the taxi price. V_i : passenger *i*'s value, i.e. the maximum willing-to-pay price. p_i : passenger *i*'s real price. U_i : passenger *i*'s utility, $U_i = V_i - p_i$.

of inconvenience attributes. In the example of Section 3, John's maximum willing-to-pay price is assumed to always be 6.58, with the in-vehicle travel time increasing from 8.5 minutes to 10 minutes.

In other scenarios, the maximum willing-to-pay price may decrease as the inconvenience degree increases. Thus, we adapt the mechanism in the second scenario, in which passengers' maximum willing-to-pay prices decrease as the inconvenience degree increases. In the second scenario, passengers can report the reduction rate of maximum willing-to-pay price in terms of the three inconvenience attributes. For example, if a passenger reports $\alpha_i^{NR} = 0.5$, it indicates that each time the number of co-riders increases by one, the maximum willing-to-pay price decreases by 0.5 dollar. Similarly, $\alpha_i^{IVT} = 0.5$ means that each time the extra in-vehicle travel time increases by 5 minutes, the maximum willing-to-pay price decreases by 0.5 dollar; $\alpha_i^{WT} = 0.5$ means that each time the extra waiting time at the transit hub increases by 5 minutes, the maximum willing-to-pay price decreases by 0.5 dollar. Thus, the three parameters α_i^{NR} , α_i^{IVT} , and α_i^{WT} represent the strictness of the requirements. The values of α_i^{NR} , α_i^{IVT} , and α_i^{WT} are given in Table 13.

The hypothetical value function is naturally presented by Formula (23).

$$V_i = V_{\max}^i - \alpha_i^{NR}NR_i - \frac{\alpha_i^{IVT}(IVT_i - t_{i0})}{5} - \frac{\alpha_i^{WT}WT_i}{5} \tag{23}$$

This value function achieves a more reasonable mechanism in which the maximum willing-to-pay price decreases as the inconvenience degree increases. Note that we use this hypothetical function just to show that our mechanism can be adapted for generalized scenarios. This form of the value function in the second scenario is less straightforward than that in the first scenario, and the reporting method may be more complex for passengers.

4.2. The results of the mechanism

We solve the model IP_0 to get the optimal matching and routing plans for the first and second scenarios, (2-3-1-0, 7-8-9-10-0, 4-5-6-0) and (2-3-1-0, 4-5-6-0, 9-10-0, 7-8-0), shown in Fig. 7(a) and (b), respectively. Table 14(a) and (b) present the price information for the two scenarios, respectively. V_{\max}^i is the taxi price, representing passenger *i*'s maximum willing-to-pay price without any inconvenience. V_i is passenger *i*'s actual maximum willing-to-pay price given the inconvenience cost caused by ridesharing considering the personalized requirement. p_i is passenger *i*'s actual paid price. The prices are all positive in both of the scenarios, indicating that the service provider receives revenue from the participants. U_i represents passenger *i*'s utility that is the maximum willing-to-pay price minus the actual paid price. All passengers' utilities are

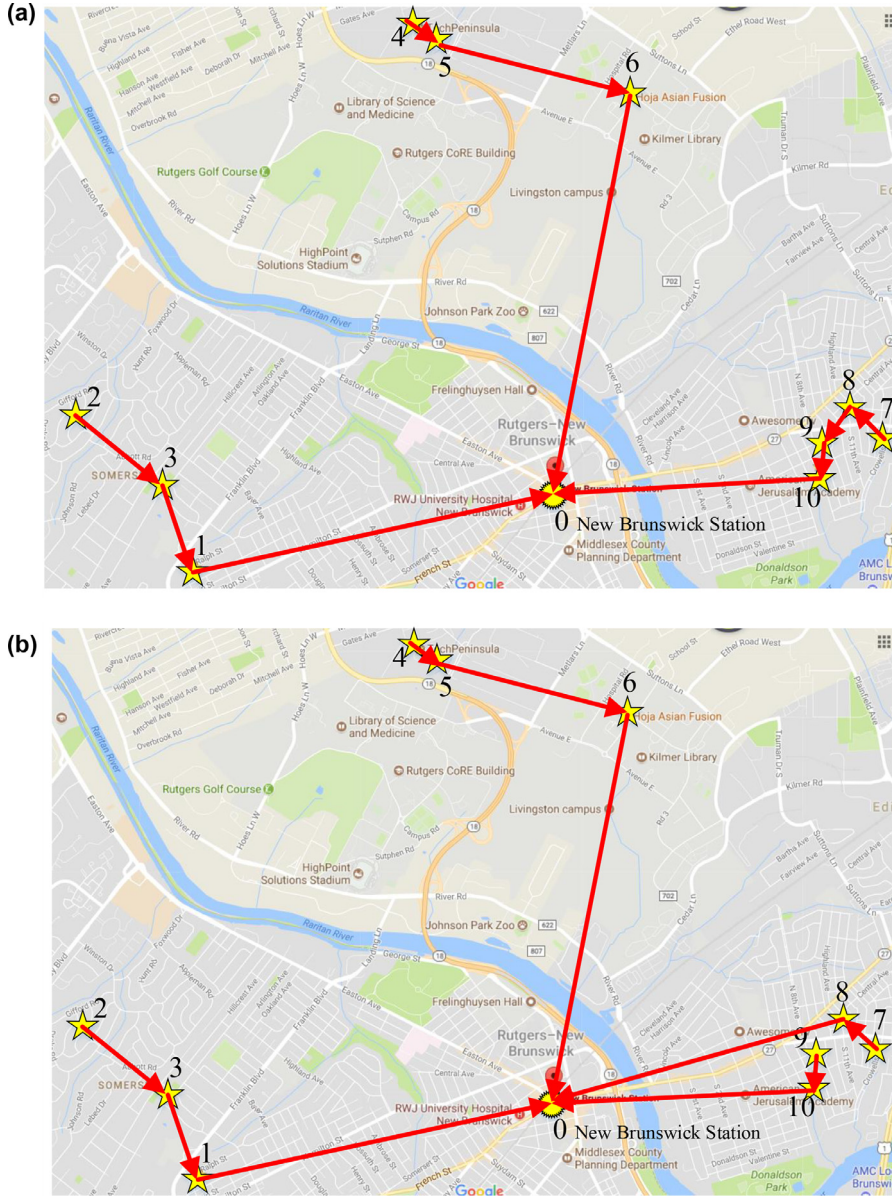


Fig. 7. (a) Optimal vehicle-passenger matching and vehicle routing plan in the first scenario. (b) Optimal vehicle-passenger matching and vehicle routing plan in the second scenario.

Algorithm 1

Obtaining the pricing mechanism.

-
- Input all parameters;
 - Solve the optimization model IP_0 and get the optimal solution $X^{IP_0^*}$, the optimal objective function value $Z_{IP_0}^*$, and each passenger's value $V_g(X^{IP_0^*})$ in $X^{IP_0^*}$;
 - For** $g = 1:n$
 - Solve the optimization model IP_g , and get the optimal objective function value $Z_{IP_g}^*$;
 - Calculate passenger(s) g 's price $p_g = Z_{IP_g}^* - (Z_{IP_0}^* - V_g(X^{IP_0^*}))$;
 - End for**
 - Output the mechanism $M(X^{IP_0^*}, \mathbf{p})$.
-

Algorithm 2

Obtaining the transition solutions $Y_g = TRS_g(X)$.

```

Input a solution  $X = \{x_{ijk}, y_{ik}, w_{ik}\}$ ;
Let  $Y_g = X$ ;
If  $NR_g > 0$ 
  Find  $k$  that  $y_{gk} = 1$ , and let  $y_{gk} = 0$ ;
  Let another vehicle  $k'$  without tasks pick up passenger(s)  $g$ ,  $y_{gk'} = 1$ ,  $w_{gk'} = 1$ , and  $x_{g0k'} = 1$ ;
  Find  $j$  that  $x_{gjk} = 1$ , and let  $x_{gjk} = 0$ ;
  If  $w_{gk} = 0$ 
    Find  $i$  that  $x_{igk} = 1$ , and let  $x_{igk} = 0$ ;
    Let  $x_{ijk} = 1$ ;
  Else
    Let  $w_{gk} = 0$ ;
    Let  $w_{jik} = 1$ ;
  End if
End if
Output  $Y_g$ .
    
```

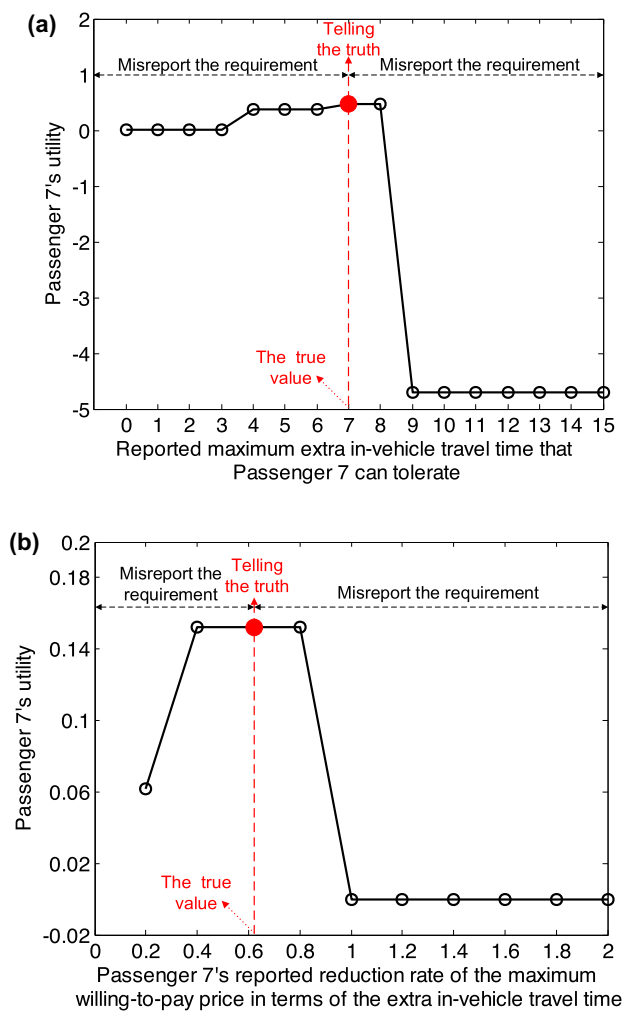


Fig. 8. (a) “Incentive compatibility” in the first scenario. (b) “Incentive compatibility” in the second scenario.

positive, indicating that all passengers are willing to choose ridesharing versus the taxi service. The non-negative utilities also indicate that the discount is able to offset passengers’ reduced maximum willing-to-pay prices caused by the inconvenience considering their personalized requirements. Furthermore, we take the 7th passenger as an example to show the property of “incentive compatibility”. Fig. 8(a) and (b) are straightforward demonstrations of “incentive compatibility” in the two scenarios, respectively. If the passenger truthfully reports the requirements on the inconvenience attributes, he will

receive no smaller utility than that if he misreports the requirements. In Fig. 8(a), we assume that the maximum extra in-vehicle travel time that passenger 7 can tolerate is 7 minutes. If the passenger truthfully reports the “7 minutes” (the red dash line), he receives the maximum utility (\$0.47) from the service. If he misreports the maximum tolerable in-vehicle travel time as being less than “7 minutes”, the price may increase and his utility will decrease. If he misreports the maximum tolerable in-vehicle travel time as being greater than “7 minutes”, the routing plan may impose him an in-vehicle travel time longer than “7 minutes”, and thereby his requirement is not satisfied. Thus, his maximum willing-to-pay price is zero but he still has to pay a price, and thus his utility is negative. Similarly, in Fig. 8(b), truthfully reporting the reduction rate (\$0.6 every five minutes) of the maximum willing-to-pay price in terms of the extra in-vehicle travel time is the optimal strategy for passenger 7. Note that Fig. 8 only presents one inconvenience attribute – extra in-vehicle travel time – as an example, and we can draw the same conclusion for the other inconvenience attributes. Finally, several previous studies (Zhao et al., 2014; Biswas et al., 2017) have considered whether the payment collected from participants can cover the transportation cost. From the results of the mechanism, the profit (the summation of all prices minus the transportation cost, $\sum_{i=1}^n p_i - TC(X^{IP_0^*})$) is \$40.74 in the first scenario and \$46.08 in the second scenario, both of which are positive. This property will be tested by a group of numerical examples with various numbers of passengers in our Part II paper.

4.3. Sensitivity analysis

Sensitivity analysis aims to investigate the dynamic process of the vehicle-passenger matching and vehicle routing plan and the prices as passengers change their requirements. We increase the strictness of one passenger's requirement on one of the three inconvenience attributes by fixing his requirements on the other two inconvenience attributes as well as all the other passengers' requirements. Fig. 9(a), (b), and (c) present how prices change due to decreasing the maximum degree of the three inconvenience attributes that the passengers can tolerate in the first scenario. Fig. 10(a), (b), and (c) show the changing process of the prices in the second scenario caused by increasing the reduction rate of maximum willing-to-pay price in terms of the increased degrees of the three inconvenience attributes, respectively.

In Fig. 9, when the maximum degree of the three inconvenience attributes that the passengers can tolerate decreases, each passenger's price either remains constant or increases. The price remains constant because passengers' changed tolerance does not impact the optimal solution of the optimization model IP_0 and the optimal vehicle-passenger matching and vehicle routing plan does not change. If the optimal matching and routing plan changes due to tightening the tolerance for the inconvenience attributes, then the passengers receive better-quality services and the price increases. Take passenger 6 in Fig. 9(a) as an example. When the maximum number of co-riders she can tolerate decreases from 3 to 2, the optimal vehicle-passenger matching and vehicle routing plan (Vehicle 1: 2-3-1-0; **Vehicle 2:** 4-5-6-0; Vehicle 3: 7-8-9-10-0) does not change and the price remains constant. When the maximum tolerable number of co-riders decreases from 2 to 1, the optimal vehicle-passenger matching and vehicle routing plan changes to “Vehicle 1: 2-3-1-0; Vehicle 2: 5-4-0; **Vehicle 3:** 6-0; Vehicle 4: 7-8-9-10-0” and the price increases due to the better-quality service. Similar conclusions are drawn from Fig. 9(b) and (c). Likewise, in Fig. 10(a), when passenger 6 increases the reduction rate of maximum willing-to-pay price in terms of the number of co-riders from \$0.4 per co-rider to \$0.6 per co-rider, the optimal vehicle-passenger matching and vehicle routing plan (Vehicle 1: 2-3-1-0; **Vehicle 2:** 4-5-6-0; Vehicle 3: 7-8-0; Vehicle 4: 9-10-0) and the price remain constant. When the reduction rate of the maximum willing-to-pay price in terms of number of co-riders is increased from \$0.6 per co-rider to \$0.8 per co-rider, the optimal vehicle-passenger matching and vehicle routing plan changes to “Vehicle 1: 2-3-1-0; Vehicle 2: 5-4-0; **Vehicle 3:** 6-0; Vehicle 4: 7-8-0; Vehicle 5: 9-10-0” and the price increases accordingly. The sensitivity analysis implies that passengers can receive higher-quality service with higher prices by placing stricter requirements on the corresponding inconvenience factors based on their preferences.

4.4. Summary

This section proposed a case study in two scenarios. The first scenario is more clear-cut. Passengers can directly report their lowest tolerance for the three inconvenience attributes. However, the first scenario has a limitation regarding the value function: it assumes that as long as one passenger's requirement is satisfied, the maximum willing-to-pay price is constant. In the second scenario, the strictness of passengers' requirements is reflected in the reduction rate of the maximum willing-to-pay price in terms of the three inconvenience attributes. The value function shows that passengers' maximum willing-to-pay prices decrease as the degree of any inconvenience attribute increases. We adopt these two scenarios to demonstrate the generality of the proposed mechanism, which is flexible enough to be adapted for different scenarios. This case study straightforwardly shows the three properties, “individual rationality”, “incentive compatibility”, and “price non-negativity” of the mechanism in the two different scenarios. Moreover, the prices collected from participants can cover the transportation cost in this case. Our Part II paper will show the service provider's profit in more cases with various numbers of participants. The sensitivity analysis demonstrates that if passengers place stricter requirements on the inconvenience attributes, they may receive higher-quality service with a higher price.

In this case study, we only use one example in two specific scenarios to interpret the results of the mechanism. The scale of the problem is small because only ten passengers are involved. Thus, this example lacks generality and is unable to test

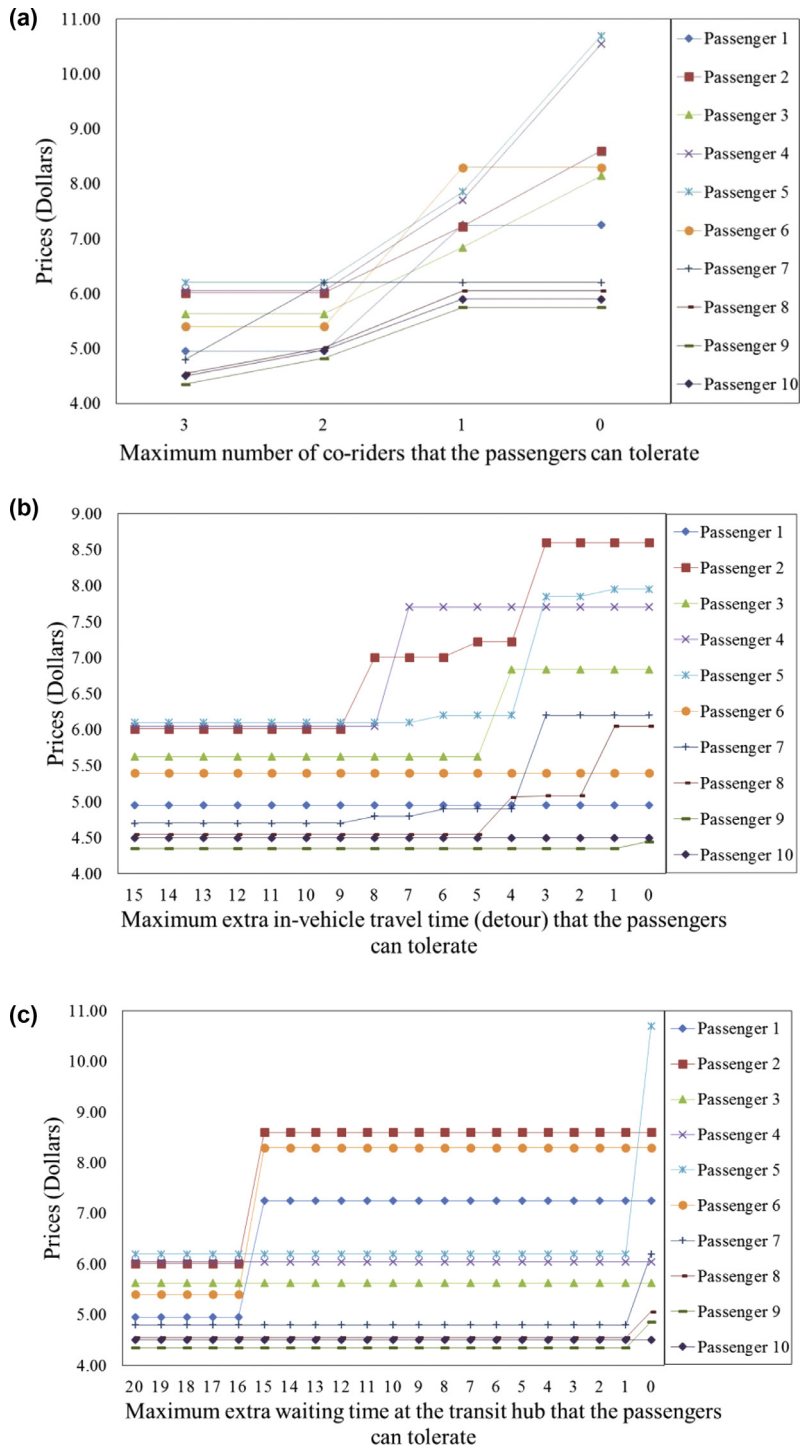


Fig. 9. (a) Price changing caused by tightening the tolerance for the number of co-riders in the first scenario. (b) Price changing caused by tightening the tolerance for extra in-vehicle travel time in the first scenario. (c) Price changing caused by tightening the tolerance for extra waiting time at the transit hub in the first scenario.

effectiveness of the potential algorithms in obtaining the mechanism for generalized large-scale problems. Our Part II paper will develop an efficient algorithm and test the performance of this algorithm using numerical examples with different scales.

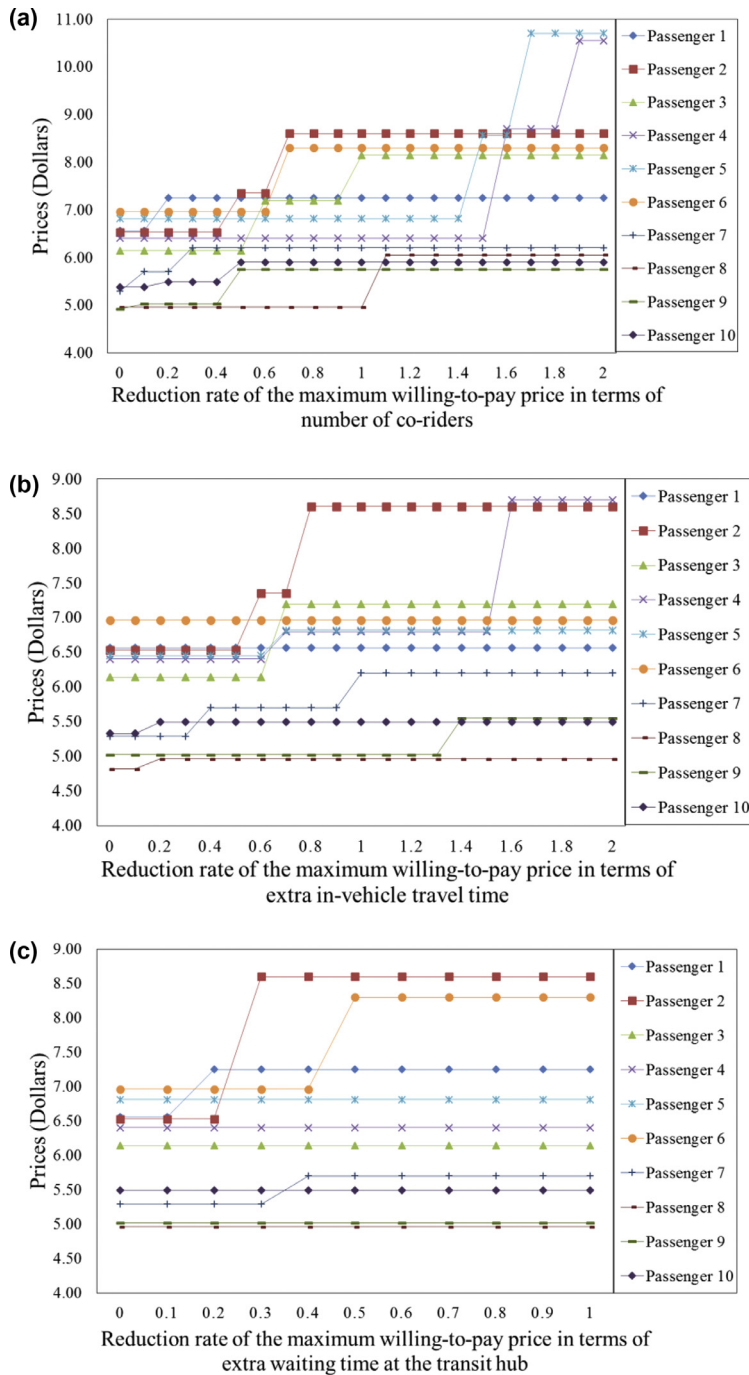


Fig. 10. (a) Price changing caused by increasing requirement strictness on the number of co-riders in the second scenario. (b) Price changing caused by increasing requirement strictness on the extra in-vehicle travel time in the second scenario. (c) Price changing caused by increasing requirement strictness on the extra waiting time at the transit hub in the second scenario.

5. Conclusions

This paper considered passengers' personalized requirements when passengers use a first-mile ridesharing service. We have designed a mechanism to incentivize passengers to participate in the ridesharing service based on their personalized requirements. This mechanism simultaneously optimizes the vehicle-passenger matching and vehicle routing plan and determines each participant's incentive price. Passengers will receive personalized service and a customized price based on

their reported personalized requirements on the inconvenience attributes, including number of co-riders, extra in-vehicle travel time, and extra waiting time at the transit hub. We proved that the proposed mechanism is individual rational, incentive compatible, and price non-negative. A case study is given to demonstrate the generality of the mechanism to different scenarios.

6. Future research

This paper has limitations. First, obtaining the mechanism is computationally challenging when the scale of the problem is large. Our Part II paper develops an effective algorithm to obtain the mechanism. Second, we only consider the static case of a first-mile ridesharing service in which all passengers book the service in advance. Our future work will consider dynamic cases in which passengers send on-demand requests. We expect to develop an online mechanism design model to incentivize passengers to reveal request information earlier, so that the algorithm can achieve system-wide optimization. Third, our future work will develop a convenient, efficient, and reasonable interactive system (e.g. smartphone APPs) that allows users to report their mobility preferences. Finally, the travel time in this paper is deterministic, while in practice it is usually uncertain. Travel time uncertainty and reliability will be considered for the mechanism design in our future work.

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Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:[10.1016/j.trb.2018.12.009](https://doi.org/10.1016/j.trb.2018.12.009).

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